Education and Culture DG
Lifelong Learning Programme

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BOOM! BOOM! - MATH AT HEART

BHAK Leibnitz, Autstria

PC based Math Project for High School Students

1 Description of the Project

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It was one of our maths lessons, when our teacher Mag. Wolfgang Narrath asked our class, if somebody wanted to participate a math project.
As two mathematically interrestet students, we agreed immediatly. Two other pupils from our class, announced for the project but we already knew that we would build a team. At our first meeting, our teacher explained the mathematical topic of the project and told us to find a adequate topic. We were very excited but it took some days of intensive inquiring to find one.
After a while we had a quite good idea. We decided to make a project about a sport topic, because we are interessted in sports. We both play soccer, which is a sport that requires a lot of endurance. This gave us the idea to test our heartrate during 5 minutes of strain and 2 minutes of recovering on a ergometer. Out of this data we were able to make our first graphs. According to the graphs. we thought, that we should fit our data with a restricted growth. As we got no satisfying results our teacher gave us the hint to shorten the measuring intervals. The second heat brought some differences especially in the first 50 seconds. We had to adjust our fitting function. We decided to choose the logistical growth for a comparison and the results went better. But not that much we hoped. Another shortening of the intervals brought along some changes but no significant ones. But we had to change the recording technique. We were not able to write down the measured data every second so we had to film the display and transcribe the values by stopping the video every second.
After the first trial we added a measurement of 2 minutes of regeneration after the resistance phase. It was just a few points in the second trial but when we shoertened the interval in the third trial, there came out huge differences and astonishing new results.
We compared the different results and found out some interesting things but unfortunately we were not able to interpret. To do so, we will have to study medicine or sports medicine.

2 Theory

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What Mathematics do we need

Open / Close

1. Functions

1. restricted growth
   $y(t) = K * \left(1 + c * a^t\right)$
2. logistic growth
   $y(t) = \frac{K}{1 + c * a^t}$
3. exponential growth
   $y(t) = c * a^t$
4. logistic decay
   $y(t) = y(t) = \frac{K}{1 + c * a^{-t}}$
5. linear decay
   y(t) = - k*t + d

2. Derivatives and equations

1. first and second derivative
   y' (t); y'' (t)
2. exponential equations

3 Developing Models

Open / Close Print

data

Open / Close

This section contains all our measured data. In order to save space during the further presentation we hide the data tables behind abbreviations. We also assigned the minimum and the maximum values to an abbreviation.

1. data 1. trial Thomas Spath resistance phase

Input ▹
```
trial1tsres =
  {{0, 75}, {30, 116}, {60, 119}, {90, 132}, {120, 135}, {150, 136},
   {180, 137}, {210, 137}, {240, 136}, {270, 139}, {300, 138}};
```

Input ▹
```
trial1tsresmin = 75;
```

Input ▹
```
trial1tsresmax = 138;
```

data 1. trial Christian Großschädl resistance phase

Input ▹
```
trial1cgres =
  {{0, 76}, {30, 122}, {60, 133}, {90, 138}, {120, 143}, {150, 145},
   {180, 146}, {210, 150}, {240, 150}, {270, 153}, {300, 156}};
```

Input ▹
```
trial1cgresmin = 76;
```

Input ▹
```
trial1cgresmax = 156;
```

3.

data 2. trial Thomas Spath resistance phase

Input ▹
```
trial2tsres = {{0, 75}, {10, 112}, {20, 120}, {30, 122}, {40, 127},
   {50, 128}, {60, 131}, {90, 140}, {120, 139}, {150, 138},
   {180, 137}, {210, 136}, {240, 138}, {270, 137}, {300, 139}};
```

Input ▹
```
trial2tsresmin = 75;
```

Input ▹
```
trial2tsresmax = 139;
```

4.

data 2. trial Christian Großschädl resistance phase

Input ▹
```
trial2cgres = {{0, 73}, {10, 90}, {20, 117}, {30, 120}, {40, 125},
   {50, 131}, {60, 134}, {90, 145}, {120, 146}, {150, 150},
   {180, 151}, {210, 151}, {240, 152}, {270, 156}, {300, 156}};
```

Input ▹
```
trial2cgresmin = 73;
```

Input ▹
```
trial2cgresmax = 156;
```

5.

data 3. trial Thomas Spath resistance phase

Input ▹
```
trial3tsres =
  {{0, 74}, {2, 74}, {4, 74}, {6, 78}, {8, 85}, {10, 92}, {12, 98},
   {14, 100}, {16, 103}, {18, 102}, {20, 102}, {22, 105}, {24, 107},
   {26, 108}, {28, 109}, {30, 109}, {32, 112}, {34, 114}, {36, 116},
   {38, 120}, {40, 121}, {42, 121}, {44, 122}, {46, 122}, {48, 123},
   {50, 126}, {52, 126}, {54, 126}, {56, 126}, {58, 127}, {60, 127},
   {65, 126}, {70, 126}, {75, 125}, {80, 125}, {85, 126},
   {90, 127}, {95, 126}, {100, 127}, {105, 127}, {110, 127},
   {115, 128}, {120, 128}, {130, 130}, {140, 131}, {150, 131},
   {160, 131}, {170, 130}, {180, 130}, {190, 130}, {200, 130},
   {210, 133}, {220, 131}, {230, 130}, {240, 131}, {250, 131},
   {260, 131}, {270, 129}, {280, 130}, {290, 132}, {300, 133}};
```

Input ▹
```
trial3tsresmin = 74;
```

Input ▹
```
trial3tsresmax = 133;
```

6. data 3. trial Christian Großschädl resistance phase

Input ▹
```
trial3cgres =
  {{0, 68}, {2, 68}, {4, 68}, {6, 69}, {8, 76}, {10, 79}, {12, 82},
   {14, 83}, {16, 95}, {18, 100}, {20, 110}, {22, 112}, {24, 112},
   {26, 114}, {28, 117}, {30, 118}, {32, 117}, {34, 118}, {36, 118},
   {38, 119}, {40, 120}, {42, 121}, {44, 124}, {46, 124}, {48, 125},
   {50, 126}, {52, 126}, {54, 126}, {56, 126}, {58, 127}, {60, 128},
   {65, 132}, {70, 135}, {75, 136}, {80, 136}, {85, 136},
   {90, 137}, {95, 138}, {100, 139}, {105, 140}, {110, 141},
   {115, 142}, {120, 142}, {130, 143}, {140, 144}, {150, 146},
   {160, 145}, {170, 145}, {180, 145}, {190, 146}, {200, 147},
   {210, 147}, {220, 147}, {230, 147}, {240, 149}, {250, 148},
   {260, 149}, {270, 149}, {280, 150}, {290, 150}, {300, 149}};
```

Input ▹
```
trial3cgresmin = 68;
```

Input ▹
```
trial3cgresmax = 149;
```

7. data 3. trial Christoph Spath resistance phase

Input ▹
```
trial3csres =
  {{0, 85}, {2, 86}, {4, 90}, {6, 94}, {8, 98}, {10, 99}, {12, 100},
   {14, 102}, {16, 105}, {18, 110}, {20, 112}, {22, 114}, {24, 116},
   {26, 116}, {28, 117}, {30, 117}, {32, 120}, {34, 121}, {36, 121},
   {38, 121}, {40, 122}, {42, 123}, {44, 123}, {46, 125}, {48, 125},
   {50, 125}, {52, 126}, {54, 126}, {56, 127}, {58, 127}, {60, 129},
   {65, 129}, {70, 130}, {75, 131}, {80, 133}, {85, 133},
   {90, 133}, {95, 134}, {100, 134}, {105, 135}, {110, 136},
   {115, 137}, {120, 138}, {130, 140}, {140, 140}, {150, 140},
   {160, 140}, {170, 140}, {180, 140}, {190, 141}, {200, 142},
   {210, 143}, {220, 144}, {230, 144}, {240, 145}, {250, 144},
   {260, 145}, {270, 147}, {280, 147}, {290, 144}, {300, 147}};
```

Input ▹
```
trial3csresmin = 85;
```

Input ▹
```
trial3csresmax = 147;
```

8. data 2. trial Thomas Spath regeneration phase

Input ▹
```
trial2tsreg = {{0, 139}, {30, 114}, {60, 103}, {90, 92}, {120, 91}};
```

Input ▹
```
trial2tsregmin = 91;
```

Input ▹
```
trial2tsregmax = 139;
```

Input ▹
```
trial2tsregtest = {{330, 114}, {360, 103}, {390, 92}, {420, 91}};
```

Input ▹
```
trial2ts = {trial2tsres, trial2tsregtest};
```

9. data 2. trial Christian Großschädl regeneration phase

Input ▹
```
trial2cgreg =
  {{0, 156}, {30, 132}, {60, 118}, {90, 102}, {120, 101}};
```

Input ▹
```
trial2cgregmin = 101;
```

Input ▹
```
trial2cgregmax = 156;
```

Input ▹
```
trial2cgregtest = {{330, 132}, {360, 118}, {390, 102}, {420, 101}};
```

Input ▹
```
trial2cg = {trial2cgres, trial2cgregtest};
```

10. data 3. trial Thomas Spath regeneration phase

Input ▹
```
trial3tsreg = {{0, 133}, {2, 133}, {4, 133}, {6, 133}, {8, 133},
   {10, 133}, {12, 133}, {14, 133}, {16, 133}, {18, 133}, {20, 132},
   {22, 130}, {24, 129}, {26, 128}, {28, 127}, {30, 124}, {32, 121},
   {34, 120}, {36, 118}, {38, 115}, {40, 112}, {42, 111},
   {44, 110}, {46, 108}, {48, 106}, {50, 106}, {52, 106},
   {54, 104}, {56, 104}, {58, 101}, {60, 102}, {65, 101},
   {70, 97}, {75, 97}, {80, 98}, {85, 99}, {90, 99}, {95, 98},
   {100, 98}, {105, 93}, {110, 91}, {115, 93}, {120, 92}};
```

Input ▹
```
trial3tsregmin = 92;
```

Input ▹
```
trial3tsregmax = 133;
```

Input ▹

```
trial3tsregtest =
  {{302, 133}, {304, 133}, {306, 133}, {308, 133}, {310, 133},
   {312, 133}, {314, 133}, {316, 133}, {318, 133}, {320, 132},
   {322, 130}, {324, 129}, {326, 128}, {328, 127}, {330, 124},
   {332, 121}, {334, 120}, {336, 118}, {338, 115}, {340, 112},
   {342, 111}, {344, 110}, {346, 108}, {348, 106},
   {350, 106}, {352, 106}, {354, 104}, {356, 104},
   {358, 101}, {360, 102}, {365, 101}, {370, 97},
   {375, 97}, {380, 98}, {385, 99}, {390, 99}, {395, 98},
   {400, 98}, {405, 93}, {410, 91}, {415, 93}, {420, 92}};
```

Input ▹

```
trial3ts = {trial3tsres, trial3tsregtest};
```

11. data 3. trial Christian Großschädl regeneration phase

Input ▹

```
trial3cgreg =
  {{0, 149}, {2, 148}, {4, 148}, {6, 146}, {8, 143}, {10, 143},
   {12, 142}, {14, 142}, {16, 142}, {18, 142}, {20, 141}, {22, 140},
   {24, 134}, {26, 132}, {28, 129}, {30, 125}, {32, 123}, {34, 122},
   {36, 119}, {38, 119}, {40, 120}, {42, 120}, {44, 119}, {46, 119},
   {48, 119}, {50, 119}, {52, 119}, {54, 117}, {56, 115},
   {58, 115}, {60, 111}, {65, 106}, {70, 106}, {75, 106},
   {80, 106}, {85, 104}, {90, 101}, {95, 101}, {100, 103},
   {105, 105}, {110, 105}, {115, 104}, {120, 100}};
```

Input ▹

```
trial3cgregmin = 100;
```

Input ▹

```
trial3cgregmax = 149;
```

Input ▹

```
trial3cgregtest =
  {{304, 148}, {306, 146}, {308, 143}, {310, 143}, {312, 142},
   {314, 142}, {316, 142}, {318, 142}, {320, 141}, {322, 140},
   {324, 134}, {326, 132}, {328, 129}, {330, 125}, {332, 123},
   {334, 122}, {336, 119}, {338, 119}, {340, 120}, {342, 120},
   {344, 119}, {346, 119}, {348, 119}, {350, 119},
   {352, 119}, {354, 117}, {356, 115}, {358, 115},
   {360, 111}, {365, 106}, {370, 106}, {375, 106},
   {380, 106}, {385, 104}, {390, 101}, {395, 101}, {400, 103},
   {405, 105}, {410, 105}, {415, 104}, {420, 100}};
```

Input ▹

```
trial3cg = {trial3cgres, trial3cgregtest};
```

12. data 3. trial Christoph Spath regeneration phase

```
trial3csreg =
  {{0, 147}, {2, 146}, {4, 146}, {6, 145}, {8, 145}, {10, 145},
   {12, 143}, {14, 140}, {16, 139}, {18, 132}, {20, 130}, {22, 129},
   {24, 128}, {26, 127}, {28, 127}, {30, 127}, {32, 126}, {34, 126},
   {36, 126}, {38, 125}, {40, 124}, {42, 123}, {44, 122}, {46, 120},
   {48, 118}, {50, 117}, {52, 116}, {54, 115}, {56, 114},
   {58, 113}, {60, 113}, {65, 112}, {70, 112}, {75, 110},
   {80, 109}, {85, 106}, {90, 104}, {95, 104}, {100, 104},
   {105, 103}, {110, 102}, {115, 102}, {120, 101}};
```

Input ▷
```
trial3csregmin = 101;
```

Input ▷
```
trial3csregmax = 147;
```

Input ▷
```
trial3csregtest =
  {{302, 146}, {304, 146}, {306, 145}, {308, 145}, {310, 145},
   {312, 143}, {314, 140}, {316, 139}, {318, 132}, {320, 130},
   {322, 129}, {324, 128}, {326, 127}, {328, 127}, {330, 127},
   {332, 126}, {334, 126}, {336, 126}, {338, 125}, {340, 124},
   {342, 123}, {344, 122}, {346, 120}, {348, 118},
   {350, 117}, {352, 116}, {354, 115}, {356, 114},
   {358, 113}, {360, 113}, {365, 112}, {370, 112}, {375, 110},
   {380, 109}, {385, 106}, {390, 104}, {395, 104}, {400, 104},
   {405, 103}, {410, 102}, {415, 102}, {420, 101}};
```

Input ▷
```
trial3cs = {trial3csres, trial3csregtest};
```

First trial (resistance) restricted growth:

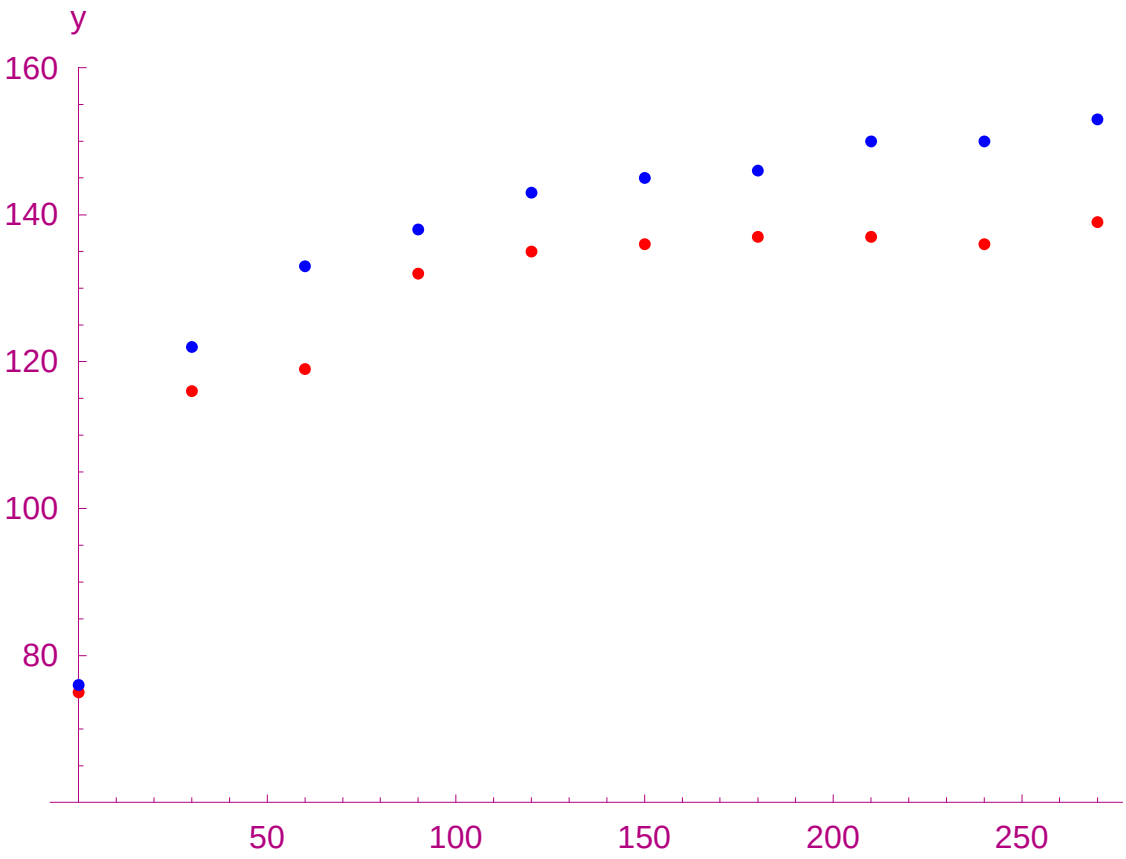
Open / Close

13. Our measured data

Input ▷
```
g1 = ListPlot[trial1tsres,
   PlotRange -> {60, 160},
   PlotStyle → {Red, PointSize[0.01]},
   DisplayFunction → Identity];
```

Input ▷
```
g2 = ListPlot[trial1cgres,
   PlotStyle → {Blue, PointSize[0.01]},
   DisplayFunction → Identity];
```

Input ▷
```
Show[g1, g2, DisplayFunction → $DisplayFunction];
```



This is a graphic of our measured data points.

14. Thomas Spath

The data points showed characteristics to fit a graph of restricted growth.

```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial1tsres,
        trial1tsresmax * (1 - c * a^x),   (* model *)
        {x}, {c, a}   (* parameters *)
    ] // Chop[#, 10^-5] &;
```

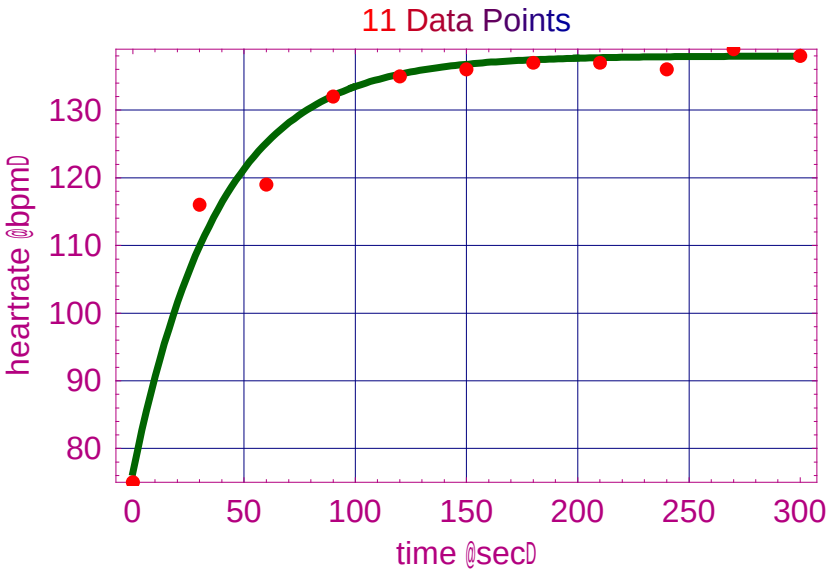
Input ▹

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial1tsres];
MDPlotFitData[trial1tsres, {fit[x]} , {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Red, PointSize[0.02], Point /@ trial1tsres},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ **Sum of Squared Error : 83.539**

$$138 - 61.5337\ 0.974242^x$$

Thisi is the first time you see the value "sum of squared errors" in our notebook, so we describe it.
The sum of squared errors is a value that shows how well a function fits compared points. It takes the distance from between the input point to the point of the function with the same x-value and squares it. The addition of this squared values calculated for each input point, is the sum of squared errors.



Additionally we calculate the average error per point in each function.

Input ▹
```
Sqrt[83.53897809026388` / 11]
```
```
2.7558
```

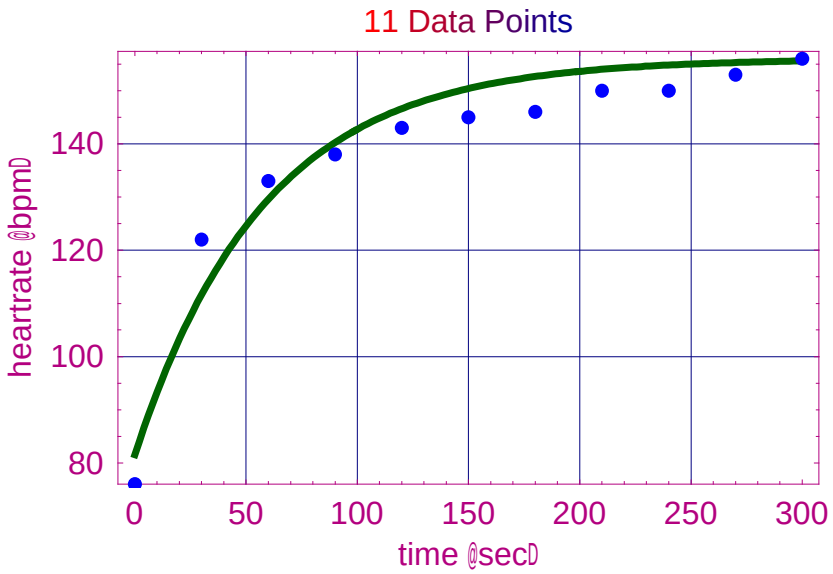
Calculated from the sum of squared errors, the average error per point is 2.7558 BPM.

15. Christian Großschädl

```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial1cgres,
        trial1cgresmax * (1 - c * a^x),      (* model *)
        {x}, {c, a}   (* parameters *)
    ] // Chop[#, 10^-5] &;
```

Input ▹
```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial1cgres];
MDPlotFitData[trial1cgres, {fit[x]}, {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Blue, PointSize[0.02], Point /@ trial1cgres},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```
$$\sum (y_i - \hat{y}_i)^2$$ Sum of Squared Error : 286.55

$$156 - 74.448\ 0.982889^{x}$$



Input ▹

```
Sqrt[286.5498578221418` / 11]
```

```
5.10392
```

Calculated from the sum of squared errors, the average error per point is 5.10392 BPM.

First trial (resistance)logistical growth:

Open / Close

16. Thomas Spath

As a comparison we fitted the same data a graph of ligistcal growth.

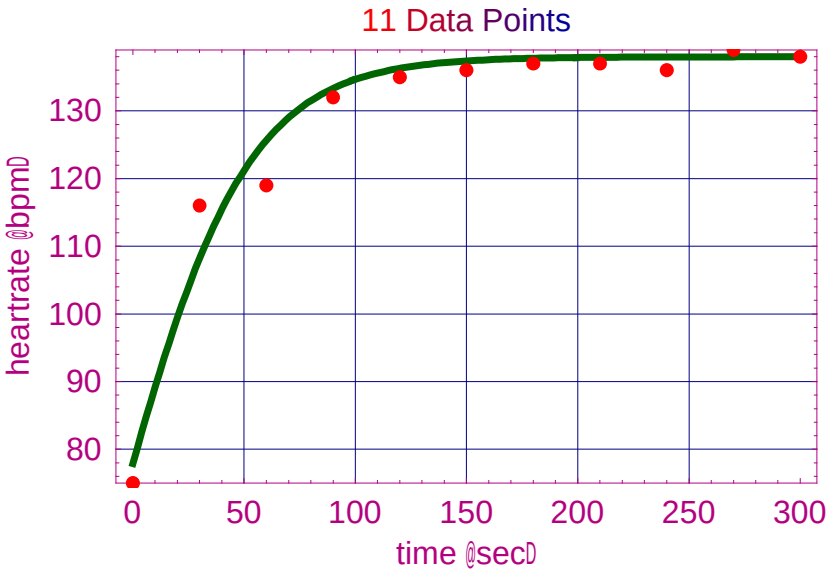
```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial1tsres,
    trial1tsresmax / (1 + b Exp[-c x]) ,    (* model *)
        {x}, {b, c}        (* parameters *)
    ] // Chop[#, 10^-15] &;
```

Input ▹

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial1tsres];
MDPlotFitData[trial1tsres, {fit[x]} , {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Red, PointSize[0.02], Point /@ trial1tsres},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$$\sum (y_i - \hat{y}_i)^2 \text{ Sum of Squared Error : } 125.125$$

$$\frac{138}{1 + 0.77162\, e^{-0.0343397\, x}}$$



Input ▹

```
Sqrt[125.12496728783721` / 11]
```

```
3.37268
```

Calculated from the sum of squared errors, the average error per point is 3.3726 BPM.

17. Christian Großschädl

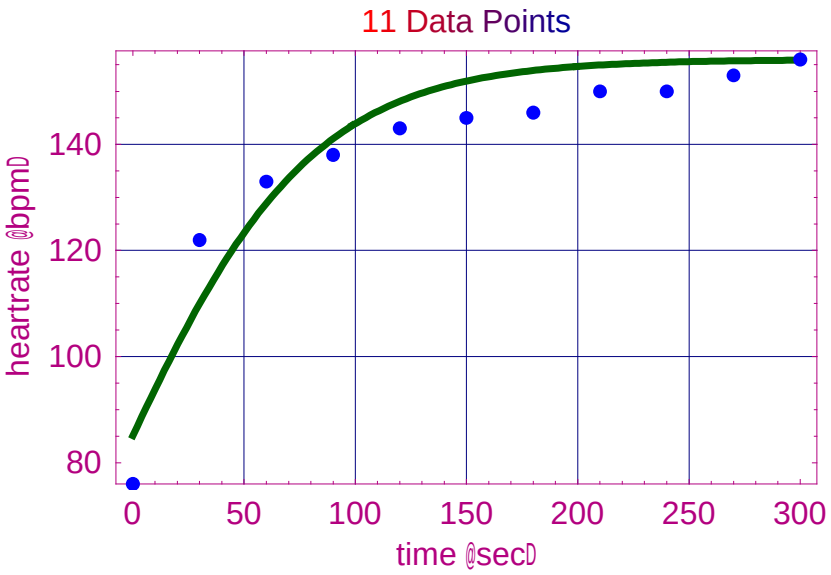
```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial1cgres,
    trial1cgresmax / (1 + b Exp[-c x]) ,   (* model *)
        {x}, {b, c}        (* parameters *)
    ] // Chop[#, 10^-15] &;
```

Input ▹

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial1cgres];
MDPlotFitData[trial1cgres, {fit[x]} , {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Blue, PointSize[0.02], Point /@ trial1cgres},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ **Sum of Squared Error : 453.662**

$$\frac{156}{1 + 0.833037\, e^{-0.0229158\, x}}$$



Input ▹
```
Sqrt[453.6620646705527` / 11]
```

```
6.42199
```

Calculated from the sum of squared errors, the average error per point is 6.4219 BPM.

Second trial (resistance)restricted growth:

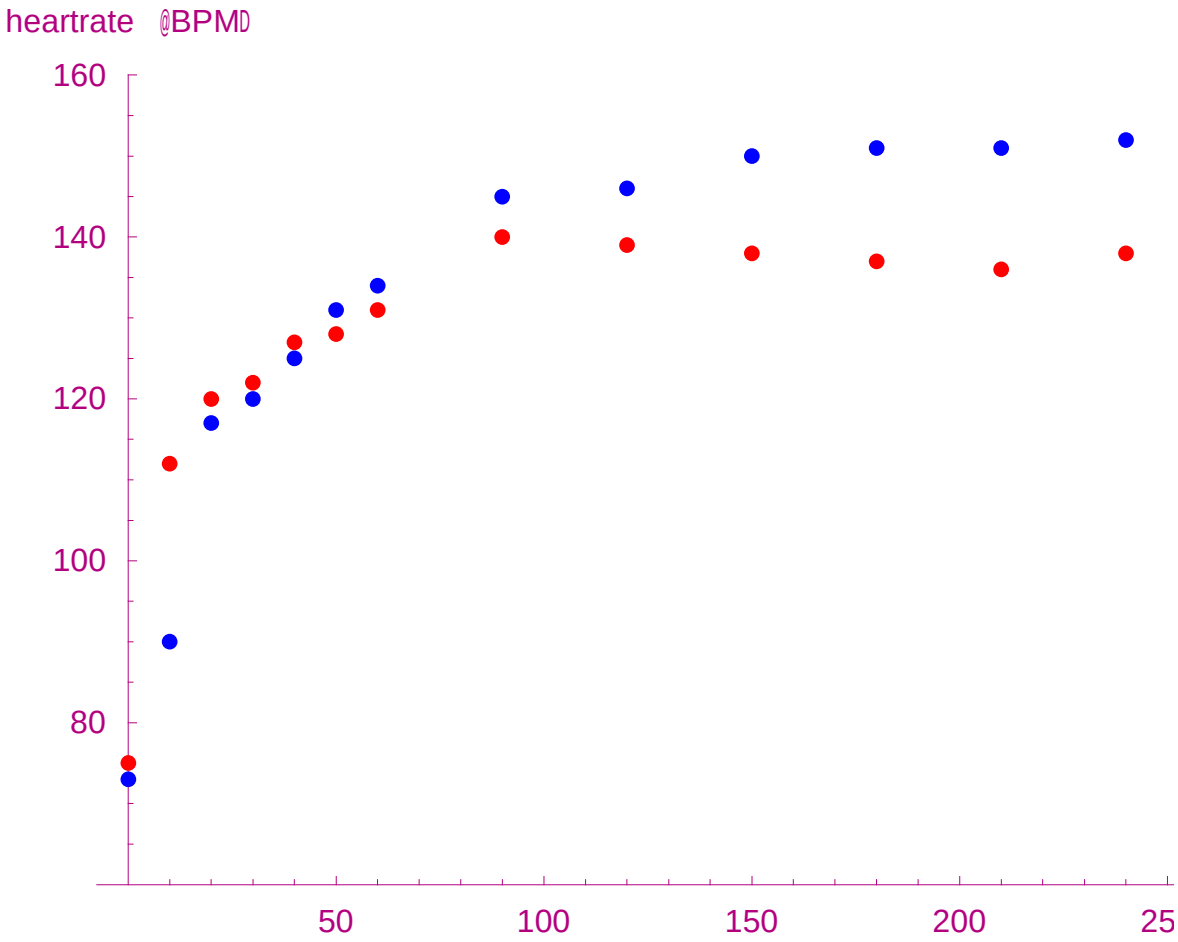
Open / Close

18. Our measured data

Input ▹
```
g1 = ListPlot[trial2tsres,
   PlotStyle → {Red, PointSize[0.012]},
   AxesLabel → {"time   [s]", "heartrate   [BPM]"},
   PlotRange → {60, 160},
   DisplayFunction → Identity];
```

Input ▹
```
g2 = ListPlot[trial2cgres,
   PlotStyle → {Blue, PointSize[0.012]},
   DisplayFunction → Identity];
```

Input ▹
```
Show[g1, g2,
  DisplayFunction → $DisplayFunction];
```



This is a graphic of our measured data points.

19. Thomas Spath

The new measured data did not bring along significant changes and so we also fitted a graph of restricted growth.

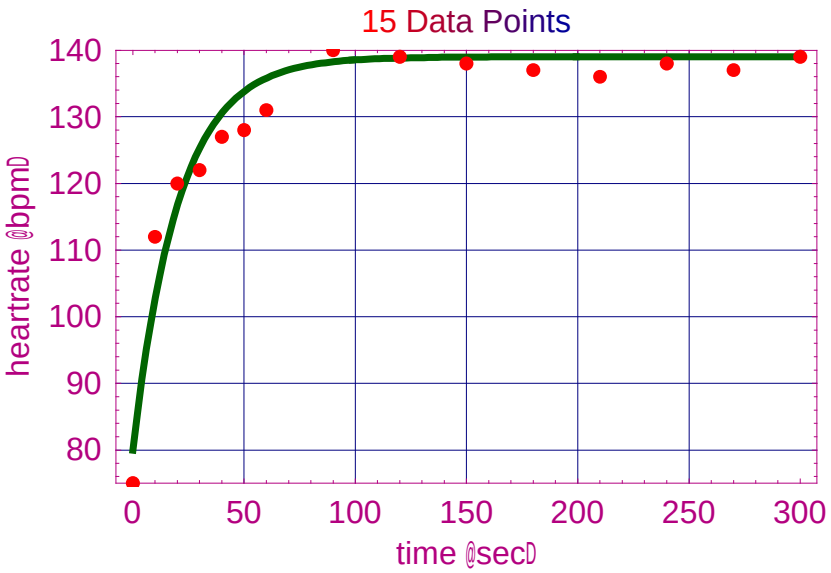
```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial2tsres,
        trial2tsresmax * (1 - c * a^x),  (* model *)
        {x}, {c, a}   (* parameters *)
    ] // Chop[#, 10^-5] &;
```

Input ▹

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial1tsres];
MDPlotFitData[trial2tsres, {fit[x]}, {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Red, PointSize[0.02], Point /@ trial2tsres},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ **Sum of Squared Error : 224.403**

$$139 - 59.0333\ 0.952555^x$$



Input ▹
```
Sqrt[224.4028956716652` / 15]
```
```
3.86784
```

Calculated from the sum of squared errors, the average error per point is 3.8678 BPM.

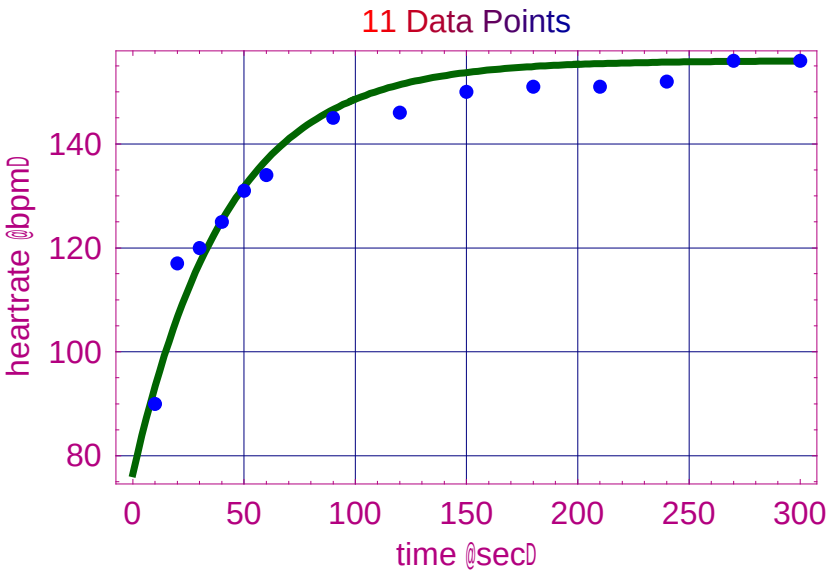
20. Christian Großschädl

```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial2cgres,
        trial2cgresmax * (1 - c * a^x),     (* model *)
        {x}, {c, a}   (* parameters *)
    ] // Chop[#, 10^-5] &;
```

Input ▹
```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial2cgres];
MDPlotFitData[trial1cgres, {fit[x]} , {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Blue, PointSize[0.02], Point /@ trial2cgres},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ **Sum of Squared Error : 411.221**

$$156 - 79.4346\ 0.976545^x$$



Input ▹
```
Sqrt[411.2205138818681` / 15]
```
```
5.2359
```

Calculated from the sum of squared errors, the average error per point is 5.235 BPM.

Second trial (resistance)logistical growth:

Open / Close

21. Thomas Spath

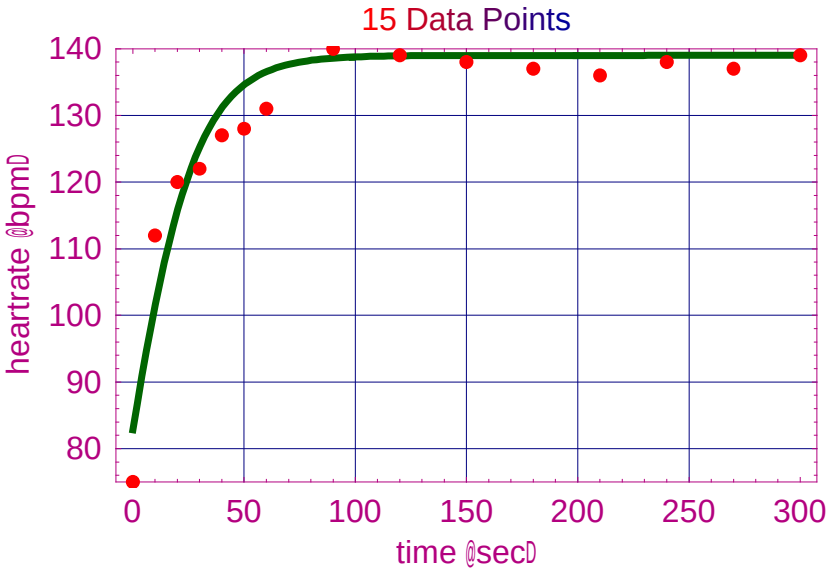
As for the first trial we fitted for comparison a graph of logistical growth.

```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial2tsres,
    trial2tsresmax / (1 + b Exp[-c x]) ,    (* model *)
        {x}, {b, c}        (* parameters *)
    ] // Chop[#, 10^-15] &;
```

Input ▹
```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial2cgres];
MDPlotFitData[trial2tsres, {fit[x]}, {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Red, PointSize[0.02], Point /@ trial2tsres},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$$\sum \left(y_i - \hat{y}_i\right)^2 \text{ Sum of Squared Error : } 313.783$$

$$\frac{139}{1 + 0.678255\, e^{-0.0605964\, x}}$$



Input ▹

```
Sqrt[313.7834027673946` / 15]
```

```
4.57372
```

Calculated from the sum of squared errors, the average error per point is 4.5737 BPM.

22. Christian Großschädl

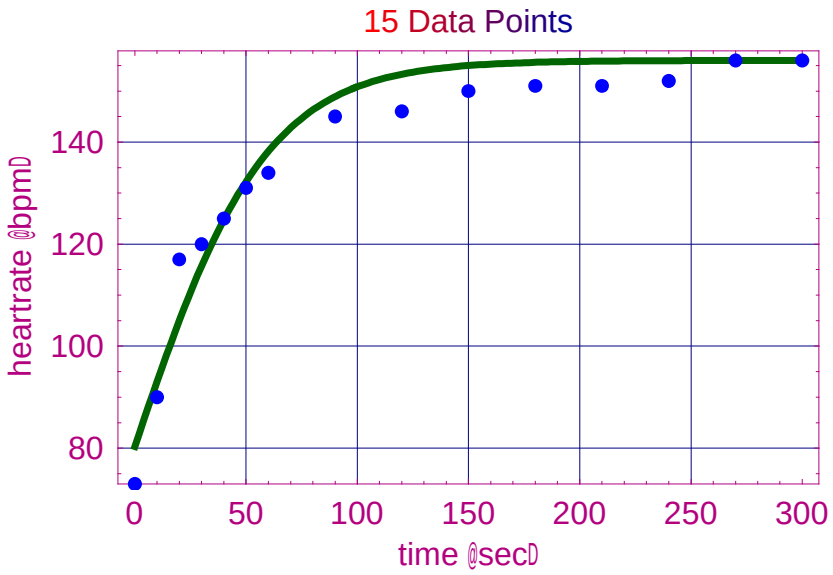
```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial2cgres,
    trial2cgresmax / (1 + b Exp[-c x]) ,   (* model *)
        {x}, {b, c}        (* parameters *)
    ] // Chop[#, 10^-15] &;
```

Input ▹

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial2cgres];
MDPlotFitData[trial2cgres, {fit[x]} , {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Blue, PointSize[0.02], Point /@ trial2cgres},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ Sum of Squared Error : 398.313

$$\frac{156}{1 + 0.94164\, e^{-0.0331291\, x}}$$



Input ▹
```
Sqrt[398.3127822578581` / 15]
```

```
5.15308
```

Calculated from the sum of squared errors, the average error per point is 5.1530 BPM.

Second trial (regeneration)linear decay:

Open / Close

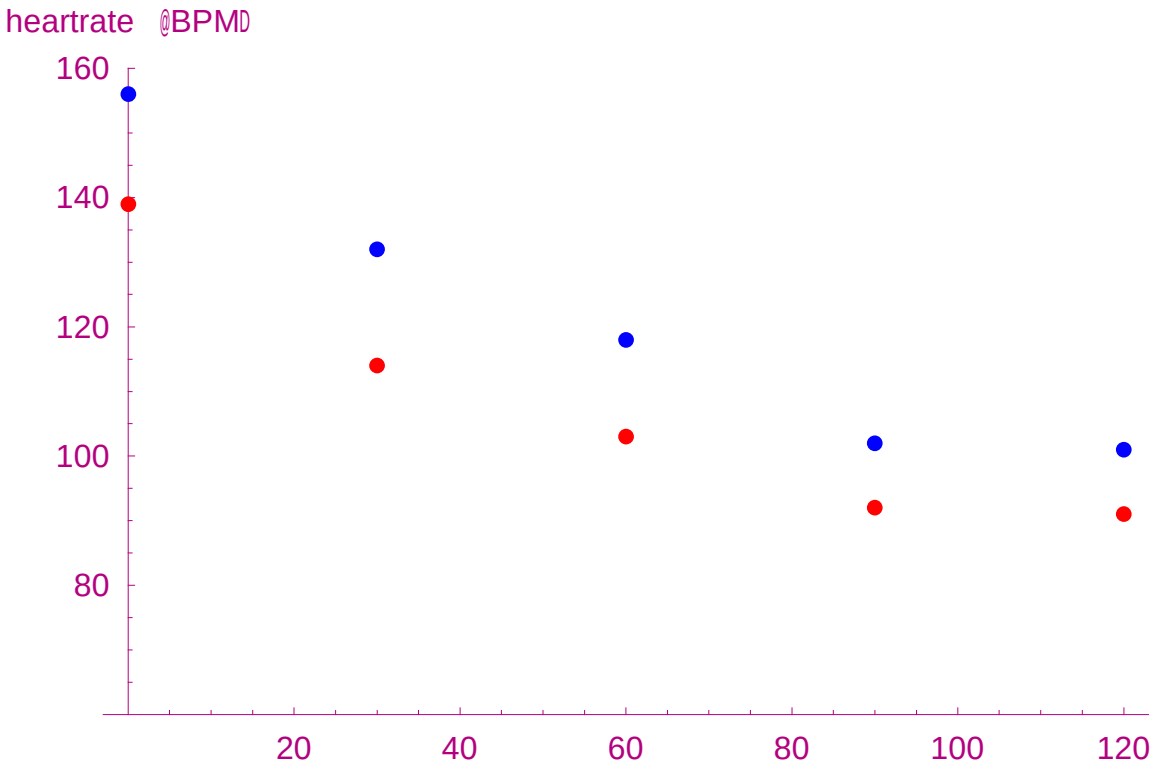
During the second trial we also measured the first 120 seconds of the regeneration phase after the resistance.

23. This is a graphic of our measured data points.

Input ▹
```
g1 = ListPlot[trial2tsreg,
   PlotStyle → {Red, PointSize[0.015]},
   AxesLabel → {"time   [s]", "heartrate   [BPM]"},
   PlotRange → {60, 160},
   DisplayFunction → Identity];
```

Input ▹
```
g2 = ListPlot[trial2cgreg,
   PlotStyle → {Blue, PointSize[0.015]},
   DisplayFunction → Identity];
```

Input ▹
```
Show[g1, g2,
  DisplayFunction → $DisplayFunction];
```



This is a graphic of our measured data points.

24. Thomas Spath

The first kind of graph we tried to fit was a linear decay.

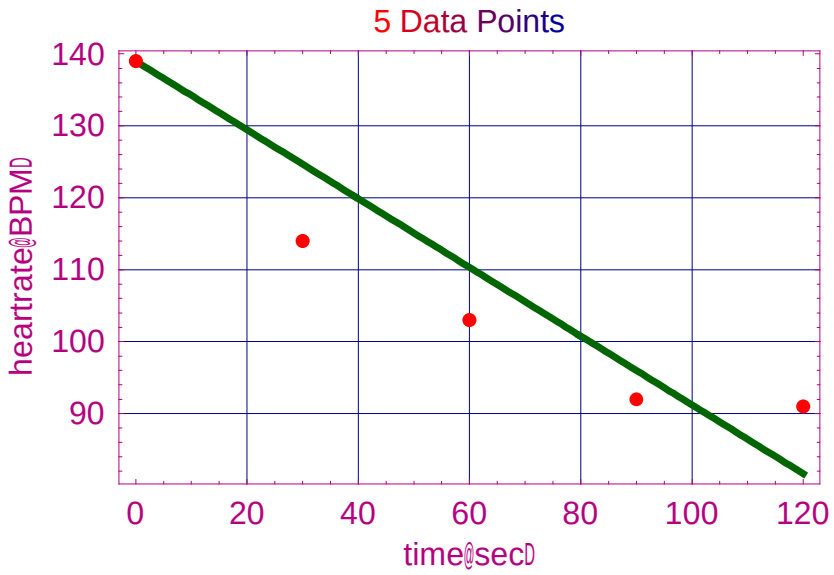
```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial2tsreg,
        -a x + trial2tsregmax,      (* model *)
        {x}, {a}    (* parameters *)
    ] // Chop[#, 10^-15] &;
```

Input ▹

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial2tsreg];
MDPlotFitData[trial2tsreg, {fit[x]}, {x, start, stop},
  FrameLabel → {"time[sec]", "heartrate[BPM]"},
  Epilog → {Red, PointSize[0.02], Point /@ trial2tsreg},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ **Sum of Squared Error : 270.667**

$$139 - 0.477778\,x$$



Input ▹

```
Sqrt[270.66666666666646` / 5]
```

```
7.35754
```

Calculated from the sum of squared errors, the average error per point is 7.3575 BPM.

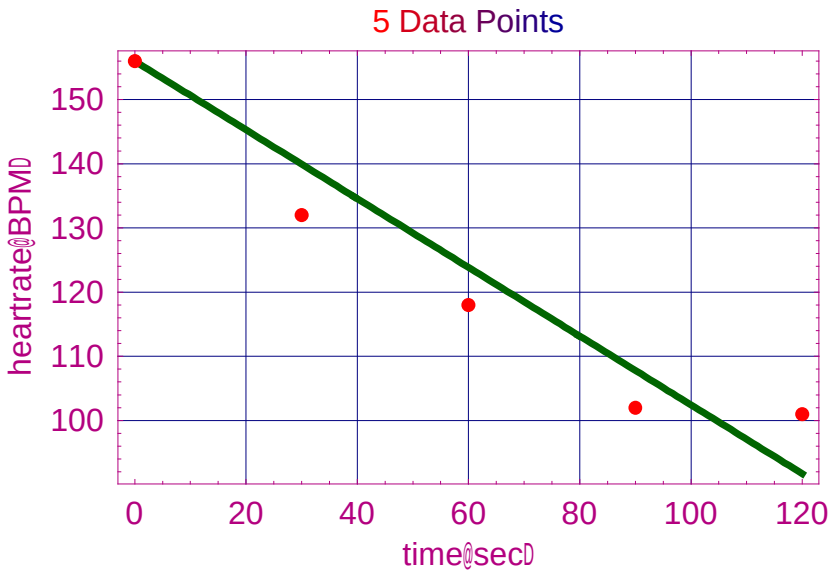
25. Christian Großschädl

Input ▹

```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial2cgreg,
        -a x + trial2cgregmax,      (* model *)
        {x}, {a}    (* parameters *)
    ] // Chop[#, 10^-15] &;

{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial2cgreg];
MDPlotFitData[trial2cgreg, {fit[x]}, {x, start, stop},
  FrameLabel → {"time[sec]", "heartrate[BPM]"},
  Epilog → {Red, PointSize[0.02], Point /@ trial2cgreg},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$$\sum \left(y_i - \hat{y}_i\right)^2$$ Sum of Squared Error : 216.867

$$156 - 0.535556\, x$$



Input ▹

```
Sqrt[216.8666666666667` / 5]
```

```
6.58584
```

Calculated from the sum of squared errors, the average error per point is 6.5858 BPM.

Second trial (regeneration)logistical decay:

Open / Close

26. Thomas Spath

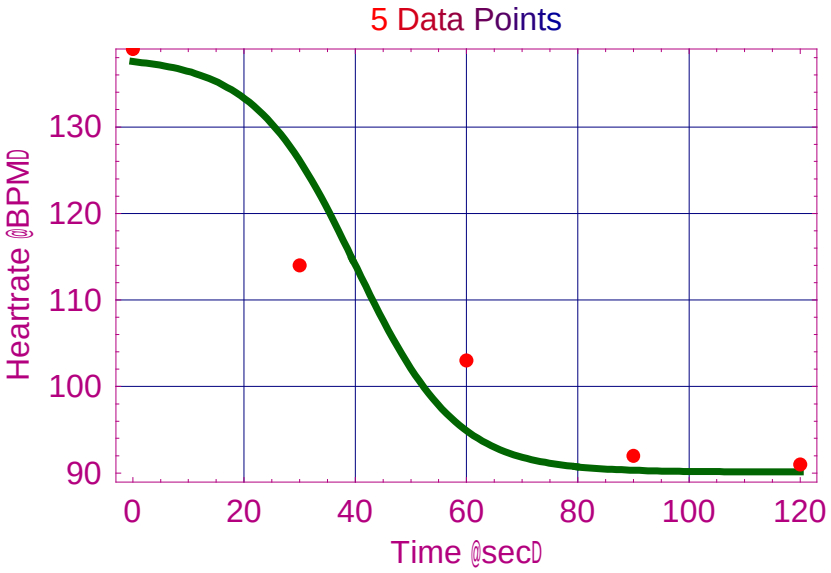
As the linear decay did optically not fit very well, we fitted a graph of logistical decay for comparison.

Input ▹

```
fit[x_] = NonlinearFit[trial2tsreg,
    f[x] = (trial2tsregmax - trial2tsregmin) / (1 + 0.01234 * e^(0.11 * x)) + d,   (* model *)
  {x}, {d,}   (* parameters *)
  ] // Chop[#, 10^-15] &;

{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial2tsreg];
MDPlotFitData[trial2tsreg, {fit[x]}, {x, start, stop},
  FrameLabel → {"Time [sec]", "Heartrate [BPM]"},
  Epilog → {Red, PointSize[0.02], Point /@ trial2tsreg},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$$\sum (y_i - \hat{y}_i)^2 \text{ Sum of Squared Error : } 217.671$$

$$90.1293 + \frac{48}{1 + 0.01234\, e^{0.11\, x}}$$



Input ▹

```
Sqrt[217.67099098741696` / 5]
```

```
6.59805
```

Calculated from the sum of squared errors, the average error per point is 6.5980 BPM.

27. Christian Großschädl

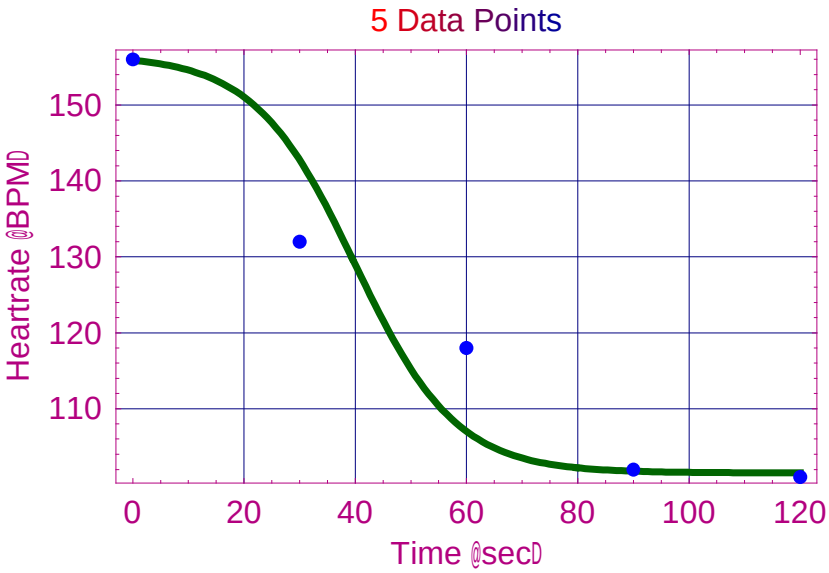
Input ▹

```
fit[x_] = NonlinearFit[trial2cgreg,

    f[x] = trial2cgregmax - trial2cgregmin / (1 + 0.01234 * e^(0.11 * x)) + d,    (* model *)

   {x}, {d,}   (* parameters *)

  ] // Chop[#, 10^-15] &;
```

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial2cgreg];
MDPlotFitData[trial2cgreg, {fit[x]}, {x, start, stop},
  FrameLabel → {"Time [sec]", "Heartrate [BPM]"},
  Epilog → {Blue, PointSize[0.02], Point /@ trial2cgreg},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$$\sum (y_i - \hat{y}_i)^2 \text{ Sum of Squared Error : } 236.948$$

$$101.552 + \frac{55}{1 + 0.01234\, e^{0.11\, x}}$$



Input ▹
```
Sqrt[236.9476540973436` / 5]
```

```
6.88401
```

Calculated from the sum of squared errors, the average error per point is 6.8840 BPM.

Second trial (regeneration)exponential decay:

Open / Close

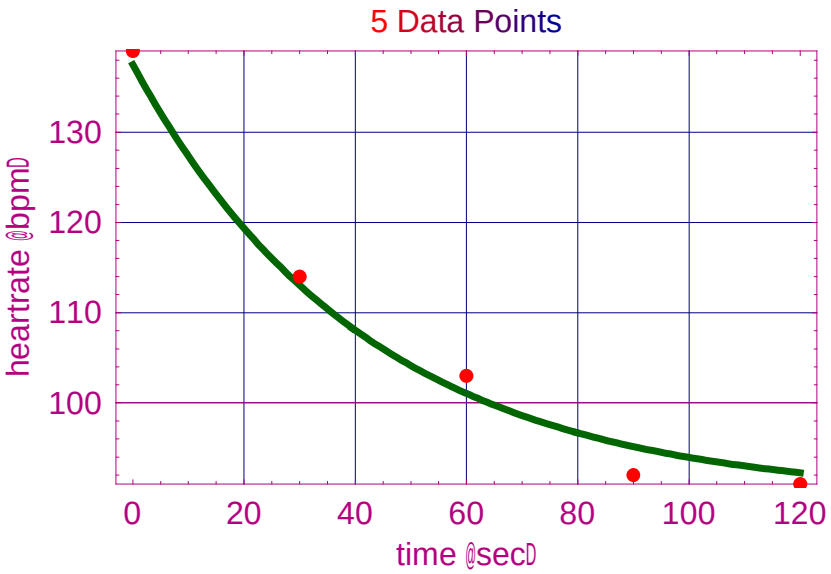
28. Thomas Spath

Due to the disappointing result of the logistical decay fit, we fitted a graph of exponential decay.

Input ▹
```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial2tsreg,
        (trial2tsregmax - trial2tsregmin) Exp[-b * 0.1 x] + c,
    (* model *)
        {x}, {b, c}      (* parameters *)
    ] // Chop[#, 10^-5] &;
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial2tsreg];
MDPlotFitData[trial2tsreg, {fit[x]}, {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Red, PointSize[0.02], Point /@ trial2tsreg},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ **Sum of Squared Error : 18.6208**

$$89.4886 + 48\, e^{-0.0237347\, x}$$



Input ▹

```
Sqrt[18.620797246034677` / 5]
```

1.92981

Calculated from the sum of squared errors, the average error per point is 1.9298 BPM.

29. Christian Großschädl

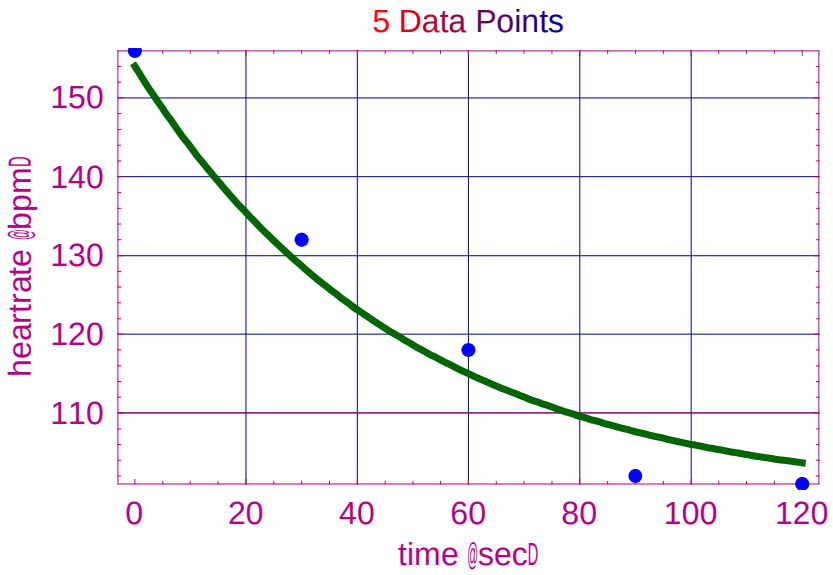
```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial2cgreg,
        (trial2cgregmax - trial2cgregmin) Exp[-b * 0.1 x] +
     c ,    (* model *)
        {x}, {b, c}      (* parameters *)
    ] // Chop[#, 10^-5] &;
```

Input ▹

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial2cgreg];
MDPlotFitData[trial2cgreg, {fit[x]} , {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Blue, PointSize[0.02], Point /@ trial2cgreg},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ **Sum of Squared Error : 62.8622**

$$99.036 + 55\, e^{-0.0206196\, x}$$



Input ▹
```
Sqrt[62.862165159683144` / 5]
```
```
3.54576
```

Calculated from the sum of squared errors, the average error per point is 3.54576 BPM.

Third trial (resistance)restricted growth:

Open / Close

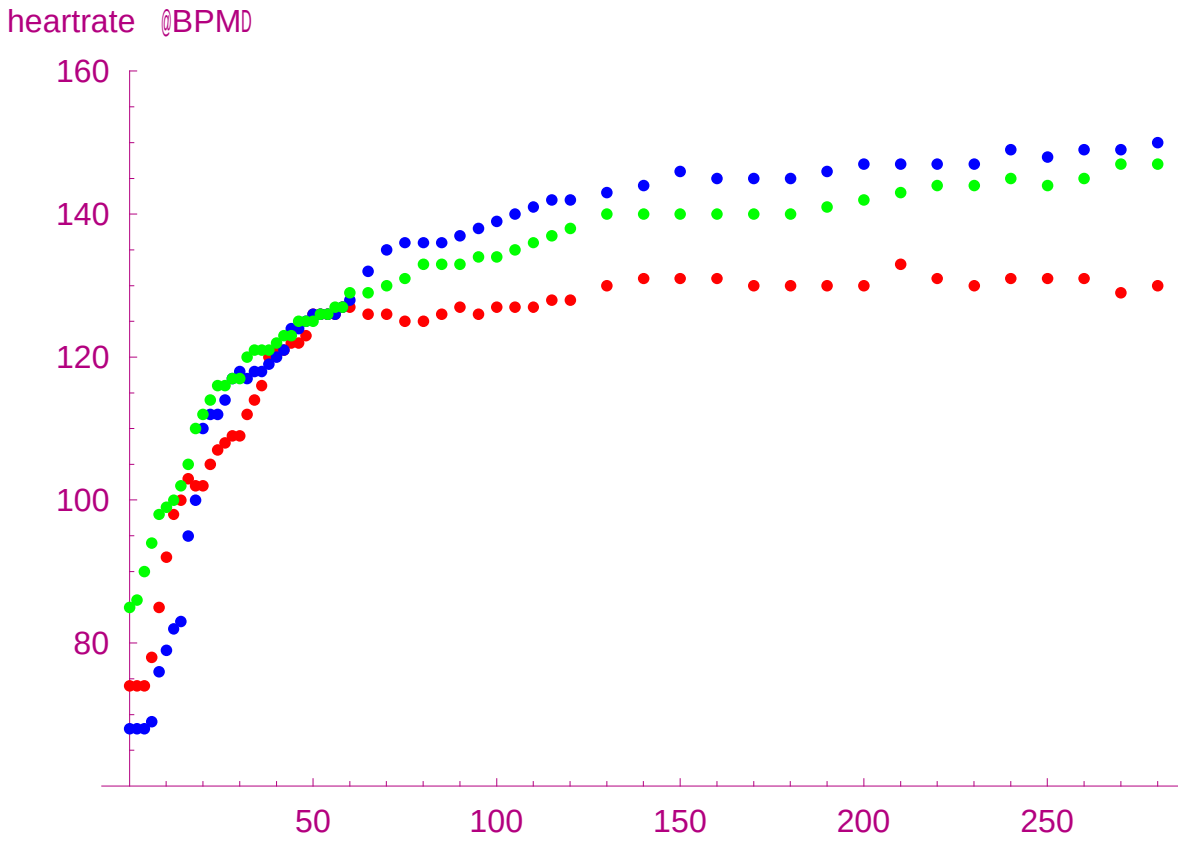
30. This is a graphic of our measured data points.

Input ▹
```
g1 = ListPlot[trial3tsres,
   PlotStyle → {Red, PointSize[0.01]},
   AxesLabel → {"time   [s]", "heartrate   [BPM]"},
   PlotRange → {60, 160},
   DisplayFunction → Identity];
```

Input ▹
```
g2 = ListPlot[trial3cgres,
   PlotStyle → {Blue, PointSize[0.01]},
   DisplayFunction → Identity];
```

Input ▹
```
g3 = ListPlot[trial3csres,
   PlotStyle → {Green, PointSize[0.01]},
   DisplayFunction → Identity];
```

Input ▹
```
Show[g1, g2, g3,
  DisplayFunction → $DisplayFunction];
```



When we compared the data of our three probands, we could see there are similarities

31. Thomas Spath

In the third trial, the most exact one, optically nothing had changed. So we went on with the well known pattern to fit with a graph of restricted growth and then compare with one of logistical growth. Additionally we got another proband to compare with.

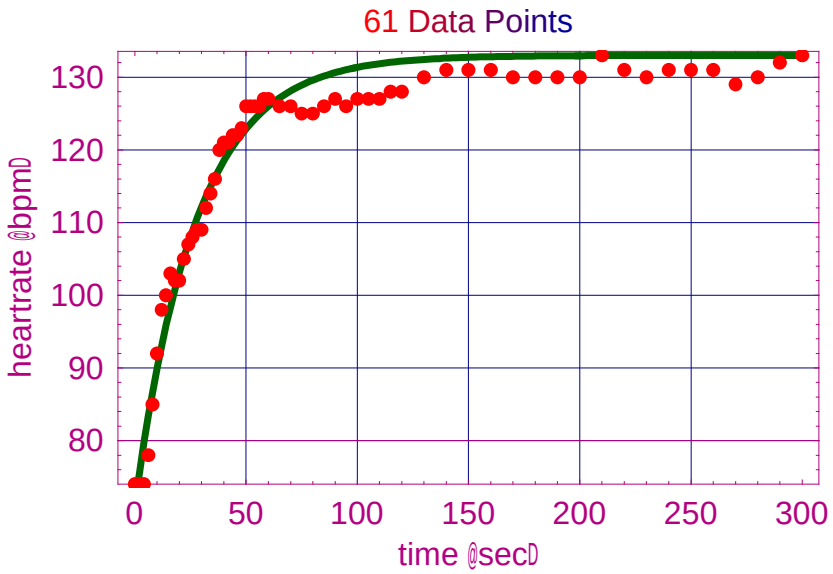
```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial3tsres,
        trial3tsresmax * (1 - c * a^x),    (* model *)
        {x}, {c, a}   (* parameters *)
    ] // Chop[#, 10^-5] &;
```

Input ▹

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial3tsres];
MDPlotFitData[trial3tsres, {fit[x]}, {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Red, PointSize[0.02], Point /@ trial3tsres},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ Sum of Squared Error : 496.631

$$133 - 61.8164\ 0.964409^{x}$$



Input ▹

```
Sqrt[496.6308617253346` / 61]
```

```
2.85333
```

Calculated from the sum of squared errors, the average error per point is 2.85333 BPM.

32. Christian Großschädl

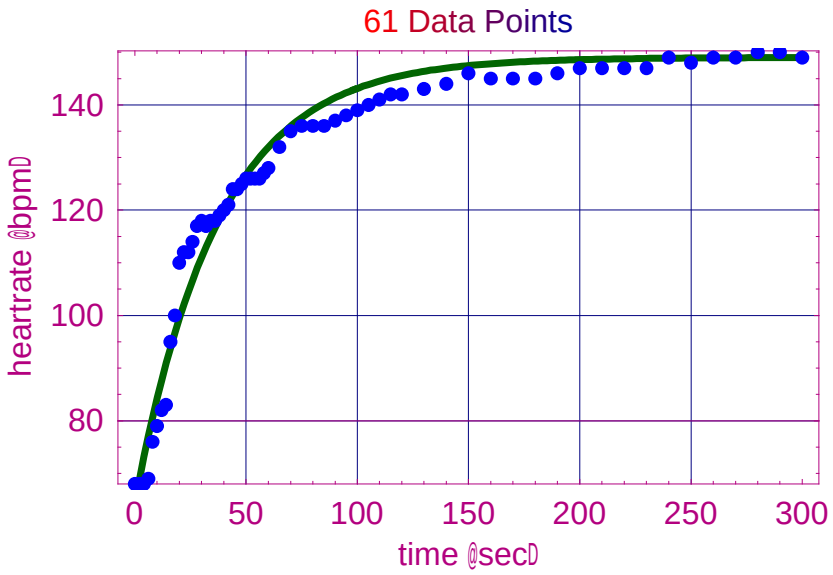
```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial3cgres,
        trial3cgresmax * (1 - c * a^x),    (* model *)
        {x}, {c, a}   (* parameters *)
    ] // Chop[#, 10^-5] &;
```

Input ▹

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial3cgres];
MDPlotFitData[trial3cgres, {fit[x]}, {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Blue, PointSize[0.02], Point /@ trial3cgres},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$$\sum \left(y_i - \hat{y}_i\right)^2 \text{ Sum of Squared Error : } 992.5$$

$$149 - 84.6003\ 0.973665^{x}$$



Input ▹
```
Sqrt[992.4996291692742` / 61]
```
```
4.03367
```

Calculated from the sum of squared errors, the average error per point is 4.03367 BPM.

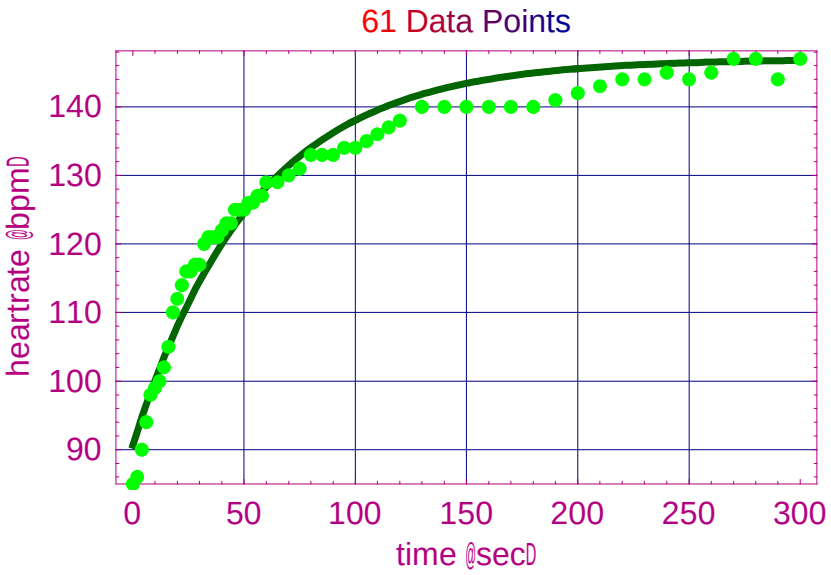
33. Christoph Spath

```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial3csres,
        trial3csresmax * (1 - c * a^x),     (* model *)
        {x}, {c, a}   (* parameters *)
    ] // Chop[#, 10^-5] &;
```

Input ▹
```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial3csres];
MDPlotFitData[trial3csres, {fit[x]} , {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Green, PointSize[0.02], Point /@ trial3csres},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ **Sum of Squared Error : 533.18**

$147 - 56.3553\ 0.981786^x$



Input ▹
```
Sqrt[533.1800949757868` / 61]
```
```
2.95646
```

Calculated from the sum of squared errors, the average error per point is 2.95646 BPM.

Third trial (resistance)logistical growth:

Open / Close

As for the first and second trial we fitted for comparison a graph of logistical growth.

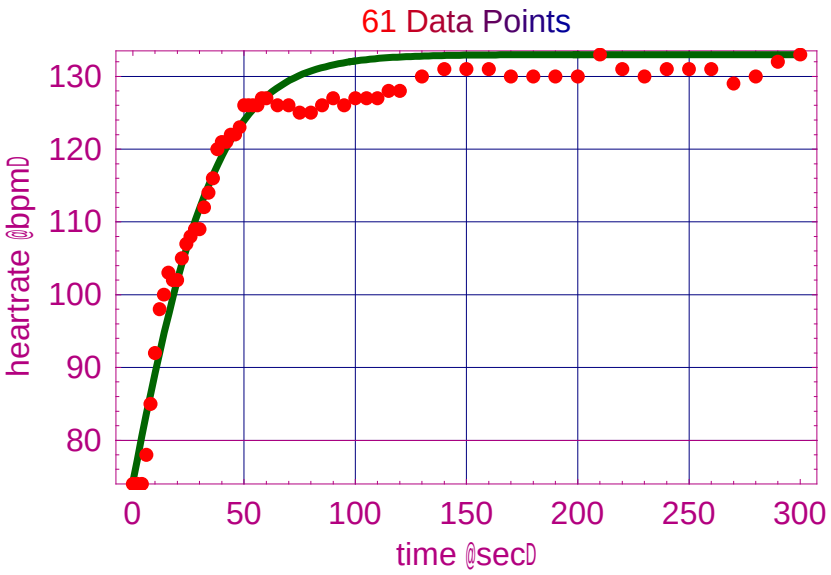
34. Thomas Spath

```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial3tsres,
    trial3tsresmax / (1 + b Exp[-c x]) ,   (* model *)
        {x}, {b, c}          (* parameters *)
    ] // Chop[#, 10^-15] &;
```

Input ▹
```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial3tsres];
MDPlotFitData[trial3tsres, {fit[x]} , {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Red, PointSize[0.02], Point /@ trial3tsres},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ **Sum of Squared Error : 617.973**

$$\frac{133}{1 + 0.790131\, e^{-0.0476525\, x}}$$



Input ▹

```
Sqrt[617.9729730138143` / 61]
```

```
3.18288
```

Calculated from the sum of squared errors, the average error per point is 3.18288 BPM.

35. Christian Großschädl

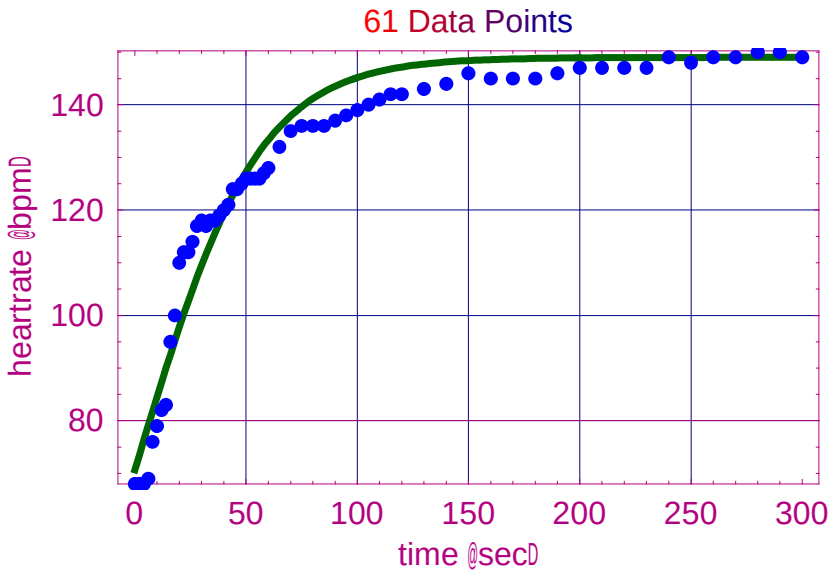
```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial3cgres,
    trial3cgresmax / (1 + b Exp[-c x]) ,    (* model *)
        {x}, {b, c}        (* parameters *)
    ] // Chop[#, 10^-15] &;
```

Input ▹

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial3cgres];
MDPlotFitData[trial3cgres, {fit[x]} , {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Blue, PointSize[0.02], Point /@ trial3cgres},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ Sum of Squared Error : 1612.97

$$\frac{149}{1 + 1.10815\, e^{-0.0374039\, x}}$$



Input ▹
```
Sqrt[1612.9703844250332` / 61]
```
```
5.14219
```

Calculated from the sum of squared errors, the average error per point is 5.14219 BPM.

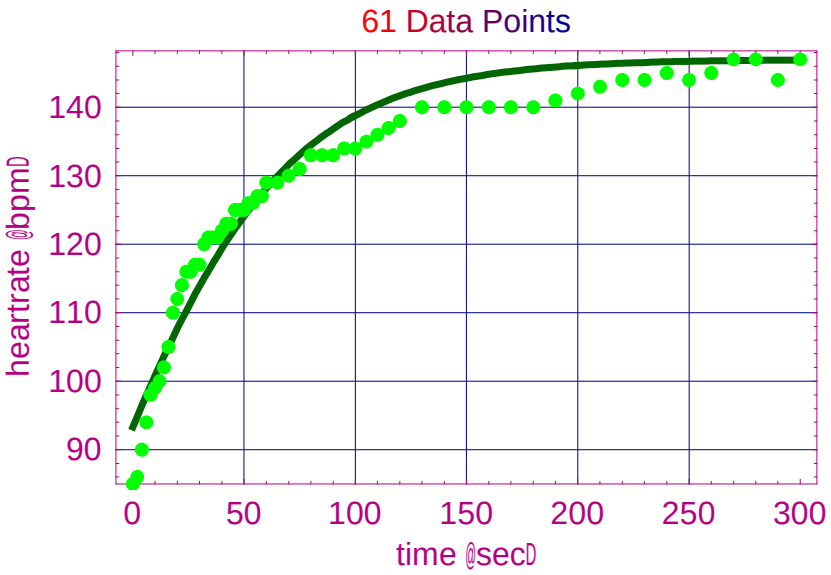
36. Christoph Spath

```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial3csres,
    trial3csresmax / (1 + b Exp[-c x]) ,    (* model *)
        {x}, {b, c}        (* parameters *)
    ] // Chop[#, 10^-15] &;
```

Input ▹
```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial3csres];
MDPlotFitData[trial3csres, {fit[x]} , {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Green, PointSize[0.02], Point /@ trial3csres},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ Sum of Squared Error : 800.573

$$\frac{147}{1 + 0.575123\, e^{-0.0227334\, x}}$$



Input ▹
```
Sqrt[800.5727566288537` / 61]
```

```
3.62273
```

Calculated from the sum of squared errors, the average error per point is 3.62273 BPM.

Third trial (regeneration)linear decay:

Open / Close

The data of the regeneration phase in trial three we fitted the same graphs as in trial two.

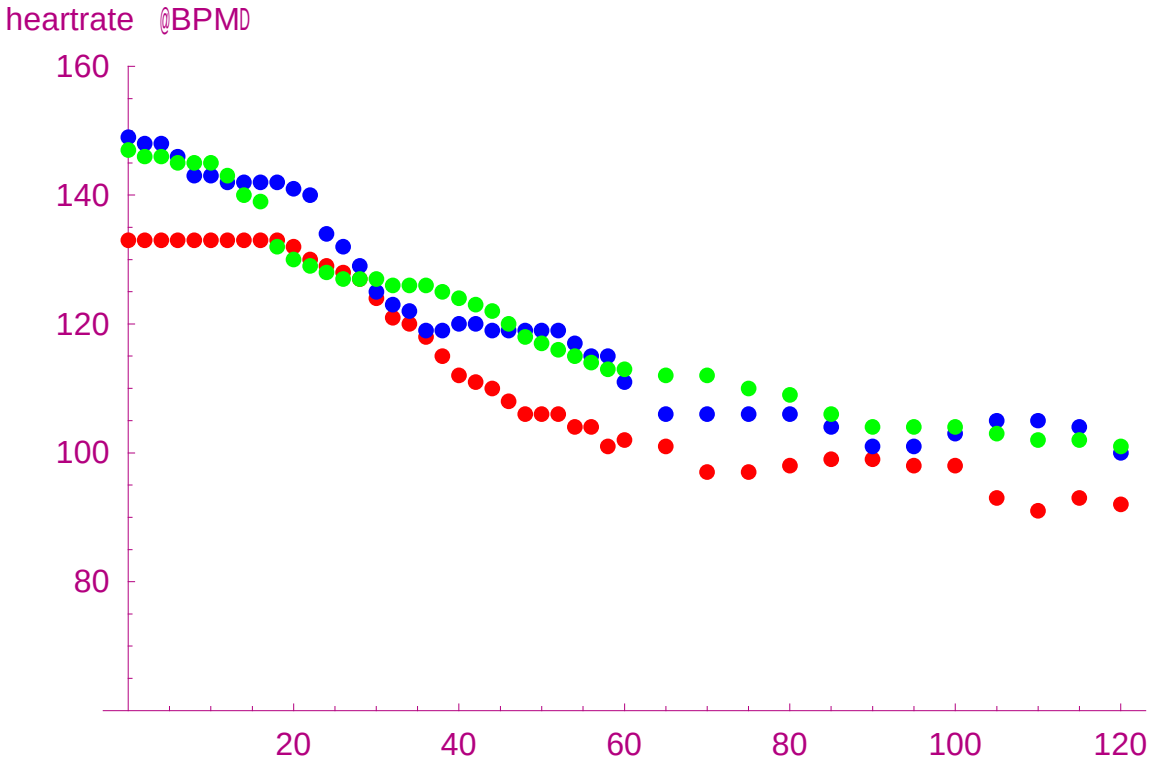
37. This is a graphic of our measured data points.

Input ▹
```
g1 = ListPlot[trial3tsreg,
   PlotStyle → {Red, PointSize[0.015]},
   AxesLabel → {"time   [s]", "heartrate   [BPM]"},
   PlotRange → {60, 160},
   DisplayFunction → Identity];
```

Input ▹
```
g2 = ListPlot[trial3cgreg,
   PlotStyle → {Blue, PointSize[0.015]},
   DisplayFunction → Identity];
```

Input ▹
```
g3 = ListPlot[trial3csreg,
   PlotStyle → {Green, PointSize[0.015]},
   DisplayFunction → Identity];
```

Input ▹
```
Show[g1, g2, g3,
  DisplayFunction → $DisplayFunction];
```



38. Thomas Spath

In this trial the additional data points brought a very different view.

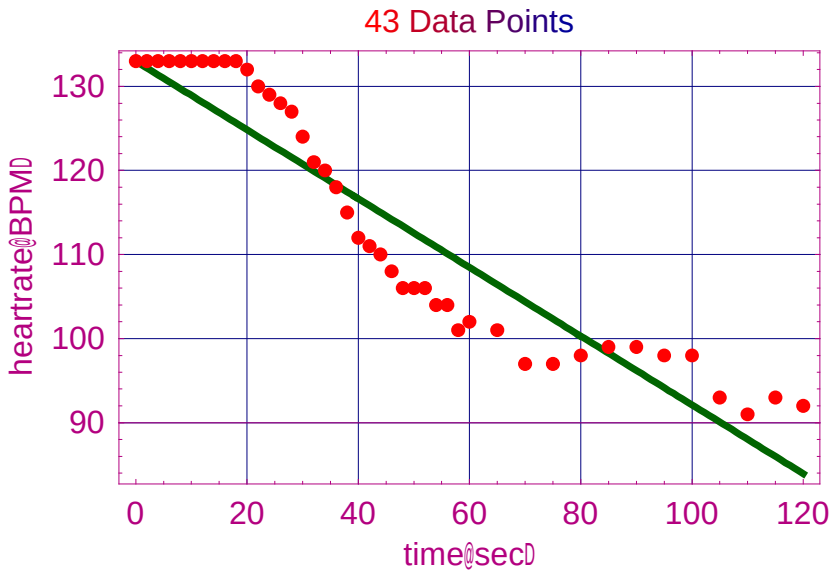
```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial3tsreg,
        -a x + trial3tsregmax,      (* model *)
        {x}, {a}    (* parameters *)
    ] // Chop[#, 10^-15] &;
```

Input ▹

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial3tsreg];
MDPlotFitData[trial3tsreg, {fit[x]}, {x, start, stop},
  FrameLabel → {"time[sec]", "heartrate[BPM]"},
  Epilog → {Red, PointSize[0.02], Point /@ trial3tsreg},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ **Sum of Squared Error : 1133.42**

$$133 - 0.408808\,x$$



Input ▹

```
Sqrt[1133.422371069619` / 43]
```

```
5.13407
```

Calculated from the sum of squared errors, the average error per point is 5.1340 BPM.

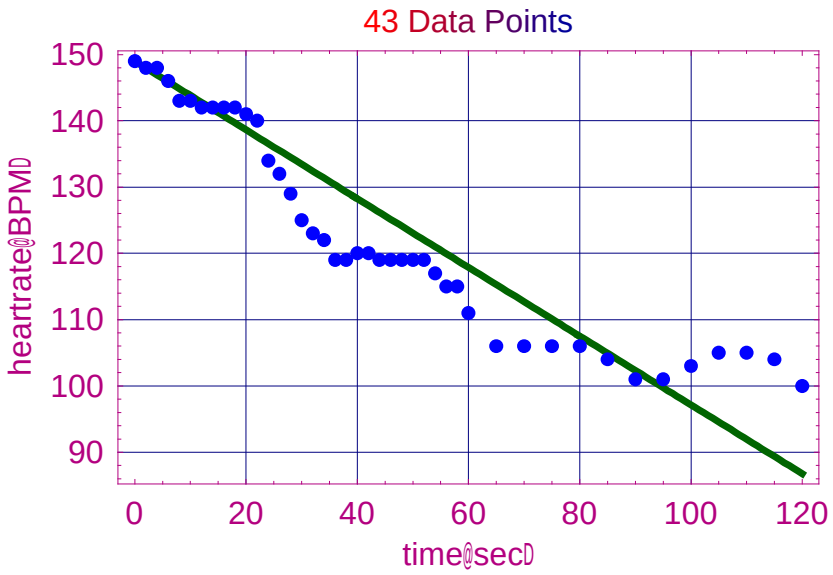
39. Christian Großschädl

Input ▹

```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial3cgreg,
        -a x + trial3cgregmax,      (* model *)
        {x}, {a}    (* parameters *)
    ] // Chop[#, 10^-15] &;

{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial3cgreg];
MDPlotFitData[trial3cgreg, {fit[x]}, {x, start, stop},
  FrameLabel → {"time[sec]", "heartrate[BPM]"},
  Epilog → {Blue, PointSize[0.02], Point /@ trial3cgreg},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$$\sum (y_i - \hat{y}_i)^2$$ Sum of Squared Error : 1777.68

$$149 - 0.518595\,x$$



Input ▹

```
Sqrt[1777.6836537794125` / 43]
```

```
6.42973
```

Calculated from the sum of squared errors, the average error per point is 6.42973 BPM.

40. Christoph Spath

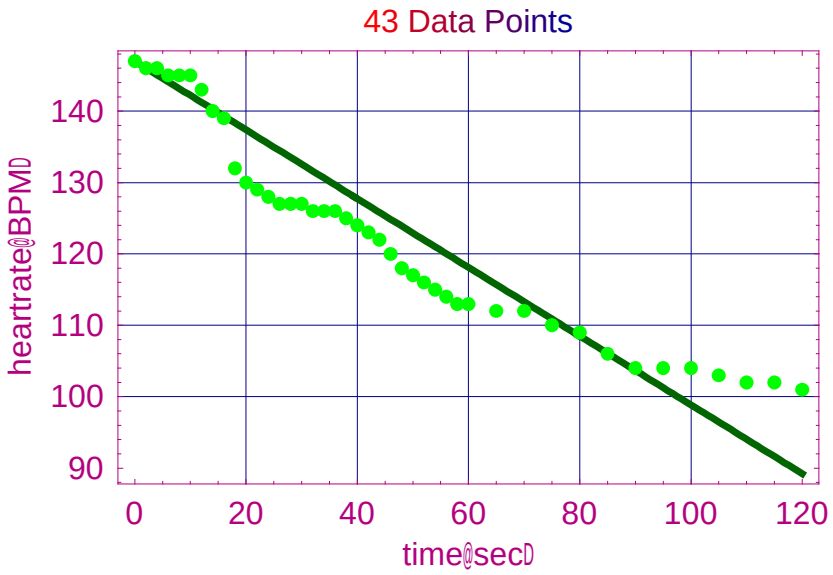
```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial3csreg,
        -a x + trial3csregmax,      (* model *)
        {x}, {a}    (* parameters *)
    ] // Chop[#, 10^-15] &;
```

Input ▹

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial3csreg];
MDPlotFitData[trial3csreg, {fit[x]}, {x, start, stop},
  FrameLabel → {"time[sec]", "heartrate[BPM]"},
  Epilog → {Green, PointSize[0.02], Point /@ trial3csreg},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$$\sum \left(y_i - \hat{y}_i\right)^2$$ **Sum of Squared Error : 1140.65**

```
147 - 0.481398 x
```



Input ▹
```
Sqrt[1140.6464565836052` / 43]
```
```
5.1504
```

Calculated from the sum of squared errors, the average error per point is 5.1504 BPM.

Third trial (regeneration)logistical decay:

Open / Close

41. Thomas Spath

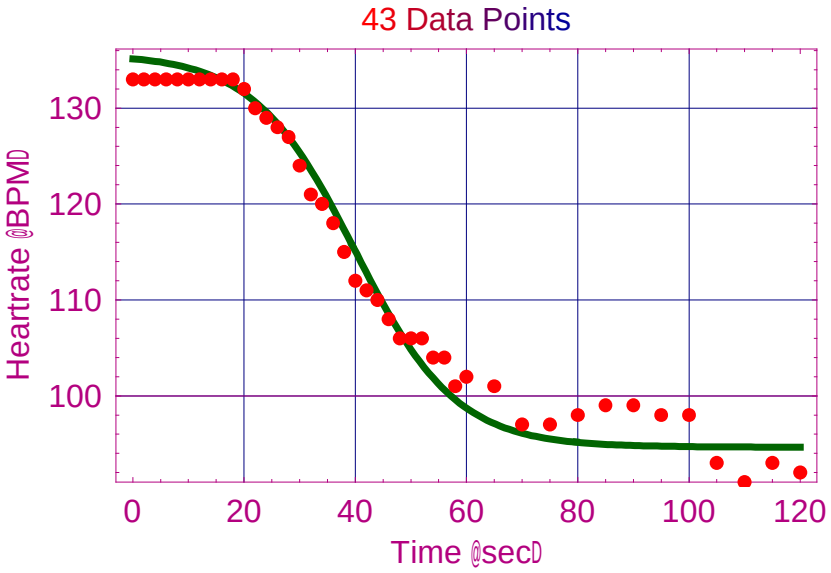
In opposite to the second trial where the linear and the logistical fit brought nearly the same results, the third trial pointed out significant differences.

Input ▹
```
fit[x_] = NonlinearFit[trial3tsreg,

    f[x] = (trial3tsregmax - trial3tsregmin) / (1 + 0.01234 * e^(0.11*x)) + d,    (* model *)

    {x}, {d}   (* parameters *)

  ] // Chop[#, 10^-15] &;
```

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial3tsreg];
MDPlotFitData[trial3tsreg, {fit[x]}, {x, start, stop},
  FrameLabel → {"Time [sec]", "Heartrate [BPM]"},
  Epilog → {Red, PointSize[0.02], Point /@ trial3tsreg},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$$\sum \left(y_i - \hat{y}_i\right)^2 \text{ Sum of Squared Error : } 199.898$$

$$94.6513 + \frac{41}{1 + 0.01234\, e^{0.11\, x}}$$



Input ▹

```
Sqrt[199.89803236364585` / 43]
```

2.15611

Calculated from the sum of squared errors, the average error per point is 2.1561 BPM.

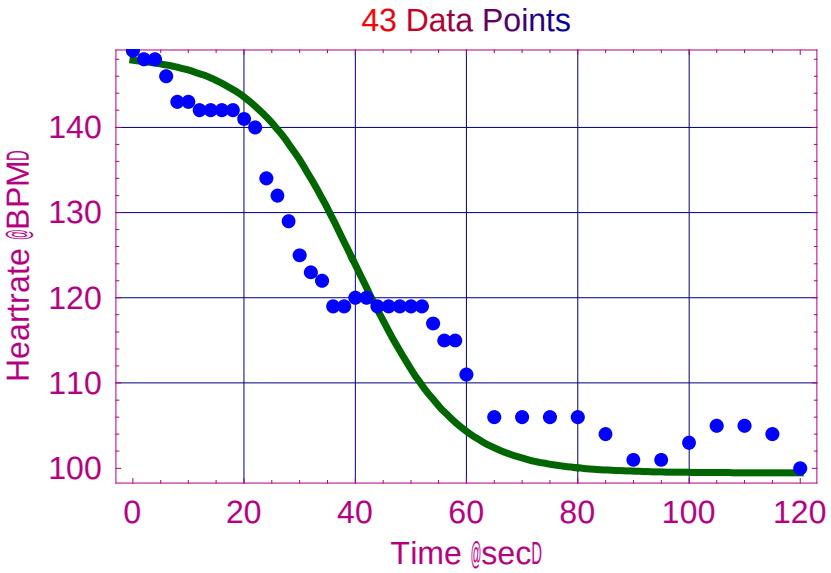
42. Christian Großschädl

Input ▹

```
fit[x_] = NonlinearFit[trial3cgreg,
    f[x] = (trial3cgregmax - trial3cgregmin)/(1 + 0.01234 * e^(0.11 * x)) + d,   (* model *)
  {x}, {d}   (* parameters *)
 ] // Chop[#, 10^-15] &;

{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial3cgreg];
MDPlotFitData[trial3cgreg, {fit[x]}, {x, start, stop},
  FrameLabel → {"Time [sec]", "Heartrate [BPM]"},
  Epilog → {Blue, PointSize[0.02], Point /@ trial3cgreg},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$$\sum \left(y_i - \hat{y}_i\right)^2 \text{ Sum of Squared Error : } 1489.33$$

$$99.4664 + \frac{49}{1 + 0.01234\, e^{0.11\, x}}$$



Input ▹

```
Sqrt[1489.3338683799288` / 43]
```

```
5.88521
```

Calculated from the sum of squared errors, the average error per point is 5.88521 BPM.

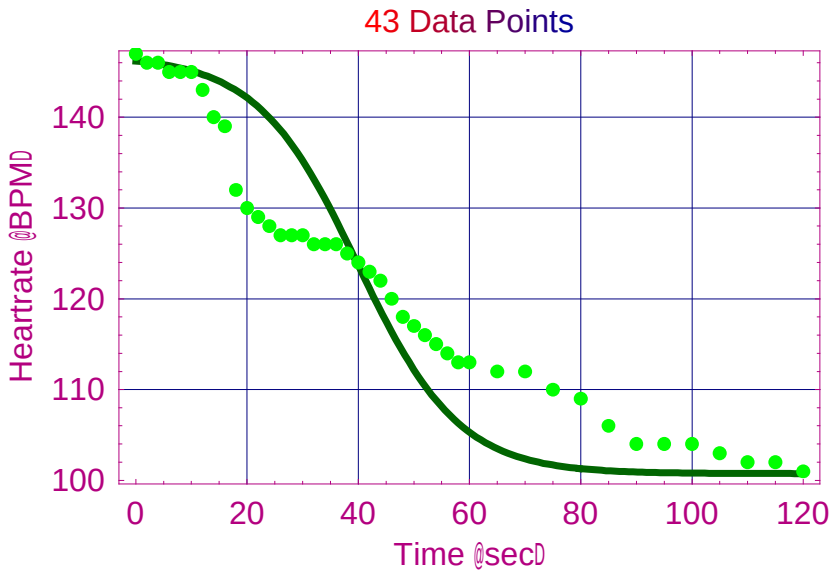
43. Christoph Spath

Input ▹

```
fit[x_] = NonlinearFit[trial3csreg,
    f[x] = (trial3csregmax - trial3csregmin)/(1 + 0.01234 * e^(0.11*x)) + d,   (* model *)
  {x}, {d}   (* parameters *)
 ] // Chop[#, 10^-15] &;

{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial3csreg];
MDPlotFitData[trial3csreg, {fit[x]}, {x, start, stop},
  FrameLabel → {"Time [sec]", "Heartrate [BPM]"},
  Epilog → {Green, PointSize[0.02], Point /@ trial3csreg},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$$\sum (y_i - \hat{y}_i)^2 \text{ Sum of Squared Error : } 1626.16$$

$$100.739 + \frac{46}{1 + 0.01234\, e^{0.11\, x}}$$



Input ▹
```
Sqrt[1626.1554416234471` / 43]
```

```
6.1496
```

Calculated from the sum of squared errors, the average error per point is 6.1496 BPM.

Third trial (regeneration)exponential decay:

Open / Close

As we did in the second trial, we also fittet an exponential decay, for showing a difference between the amout of mesured data.

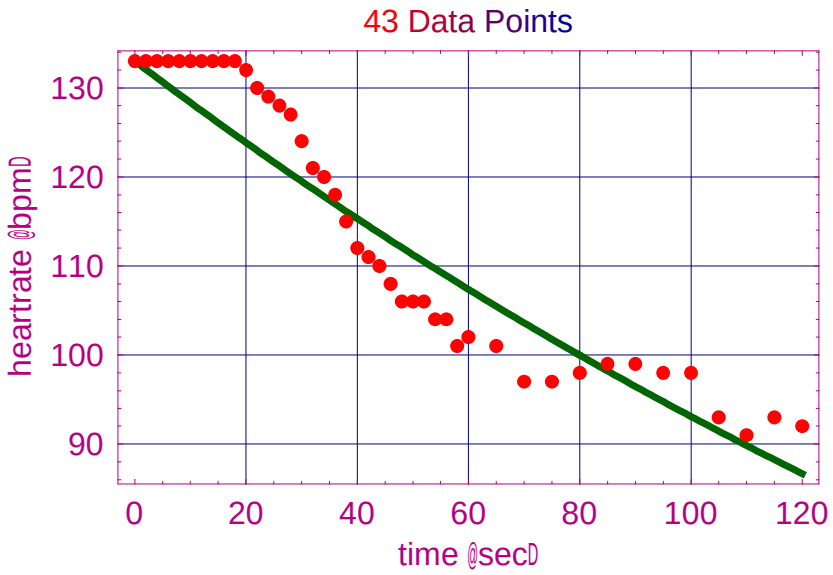
44. Thomas Spath

```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial3tsreg,
        trial3tsregmax Exp[-b x] ,   (* model *)
        {x}, {b}       (* parameters *)
    ] // Chop[#, 10^-5] &;
```

Input ▹
```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial3tsreg];
MDPlotFitData[trial3tsreg, {fit[x]}, {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Red, PointSize[0.02], Point /@ trial3tsreg},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$$\sum \left(y_i - \hat{y}_i\right)^2 \text{ Sum of Squared Error : } 999.213$$

$$133\, e^{-0.00356822\, x}$$



Input ▹

```
Sqrt[999.212675983066` / 43]
```

```
4.82053
```

Calculated from the sum of squared errors, the average error per point is 4.8205 BPM.

45. Christian Großschädl

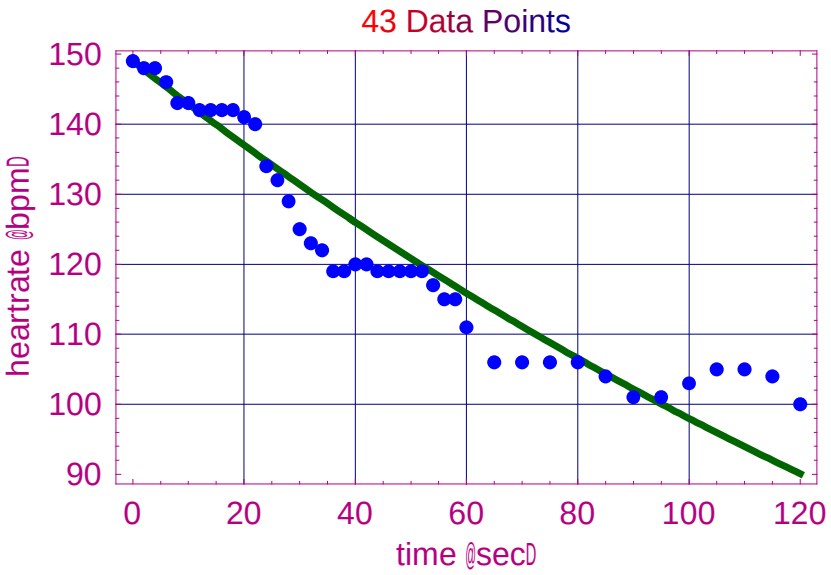
```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial3cgreg,
        trial3cgregmax Exp[-b x] ,    (* model *)
        {x}, {b}      (* parameters *)
    ] // Chop[#, 10^-5] &;
```

Input ▹

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial3cgreg];
MDPlotFitData[trial3cgreg, {fit[x]} , {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Blue, PointSize[0.02], Point /@ trial3cgreg},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ Sum of Squared Error : 1083.8

$149\, e^{-0.00419156\, x}$



Input ▹

```
Sqrt[1083.8035785178681` / 43]
```

```
5.02043
```

Calculated from the sum of squared errors, the average error per point is 5.0204 BPM.

46. Christoph Spath

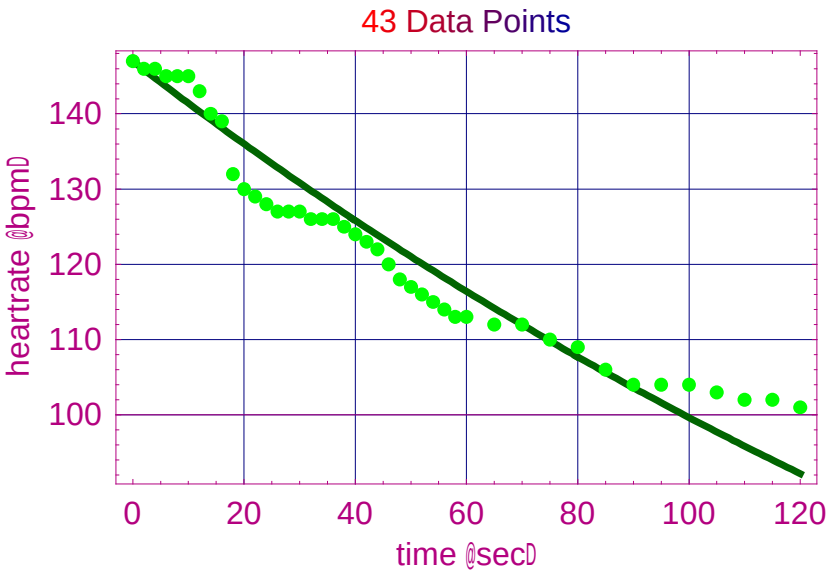
```
(*Fit data*) Clear[x, a, b, c, d, e, f, g, h];
fit[x_] = NonlinearFit[trial3csreg,
        trial3csregmax Exp[-b x] ,    (* model *)
        {x}, {b}      (* parameters *)
    ] // Chop[#, 10^-5] &;
```

Input ▹

```
{start, stop} =
  {Min[#], Max[#]} &[First@Transpose@trial3csreg];
MDPlotFitData[trial3csreg, {fit[x]} , {x, start, stop},
  FrameLabel → {"time [sec]", "heartrate [bpm]"},
  Epilog → {Green, PointSize[0.02], Point /@ trial3csreg},
  PlotStyle → {{DarkGreen, Thickness[0.01]}}];
```

$\sum (y_i - \hat{y}_i)^2$ Sum of Squared Error : 636.502

$147\, e^{-0.00388772\, x}$



Input ▹
```
Sqrt[636.5016787276866` / 43]
```

```
3.84738
```

Calculated from the sum of squared errors, the average error per point is 3.8473 BPM.

Caution!!! Steep Hill

Open / Close

On our trip to Germany, with the support of Uta, we constructed an example of the use of differential calculations. But we did not calculate the value of a steep hill sign, where it is also used. We took one of our functions to calculate and plot an average and an instaneous rate of change for it.

47. Example for Average Rate of Change

Input ▹
```
f[x_] = 94.65130862027519` + 41 / (1 + 0.01234` e^(0.11` x));
```

Input ▹
```
m = (f[80] - f[40]) / 40
```

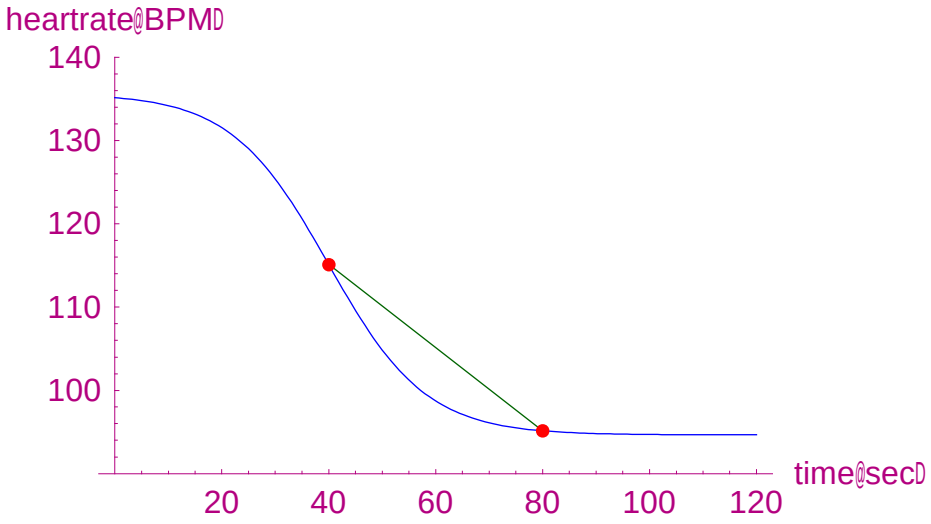
```
-0.498826
```

This is our average rate of change. That means that we have an average incline of 49.88 %.

Input ▹
```
sekante = {DarkGreen, Line[{{80, f[80]}, {40, f[40]}}]};
```

Input ▹
```
points =
  {Red, PointSize[0.02], Point /@ {{40, f[40]}, {80, f[80]}}};
```

Input ▹
```
Plot[f[x], {x, 0, 120}, PlotStyle → Blue,
  AxesLabel → {"time[sec]", "heartrate[BPM]"},
  Epilog → {sekante, points}, PlotRange → {90, 140}];
```



Here you can see the secant which traverses the function in the two red points.

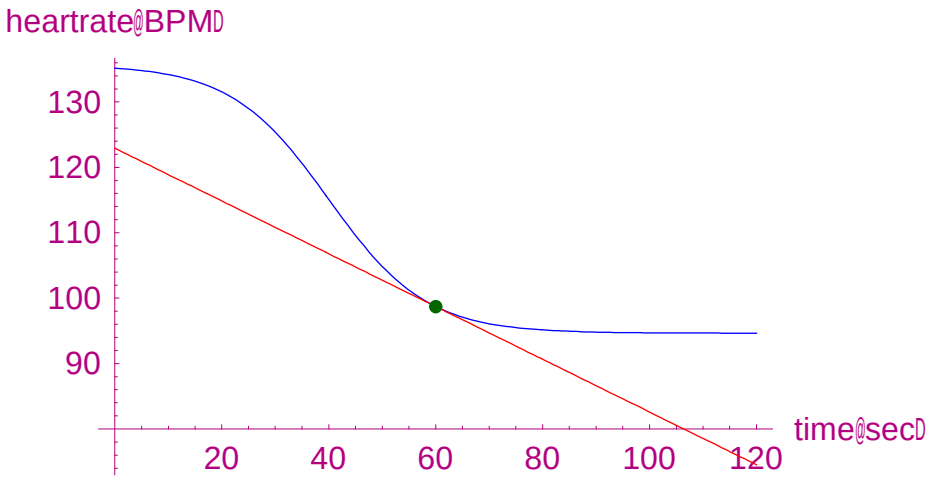
48. instaneous rate of change third trial Thomas Spath

Input ▹
```
k = f'[60]
```

```
-0.403351
```

This would be the difference if we raised the x value by 1.
This value shows the change at the point f [60].

```
Input ▷  tg[x_] = k * x + d;

Input ▷  glg1 = tg[60] == f[60];

Input ▷  NSolve[glg1, d];

Input ▷  d = 122.92346238325055`;

Input ▷  Plot[{f[x], tg[x]}, {x, 0, 120}, PlotStyle → {Blue, Red},
           AxesLabel → {"time[sec]", "heartrate[BPM]"},
           Epilog → {DarkGreen, PointSize[0.02], Point[{60, f[60]}]}];
```



This is a graph of the tangent and the traversing point.

4 Result and Summary

Open / Close Print

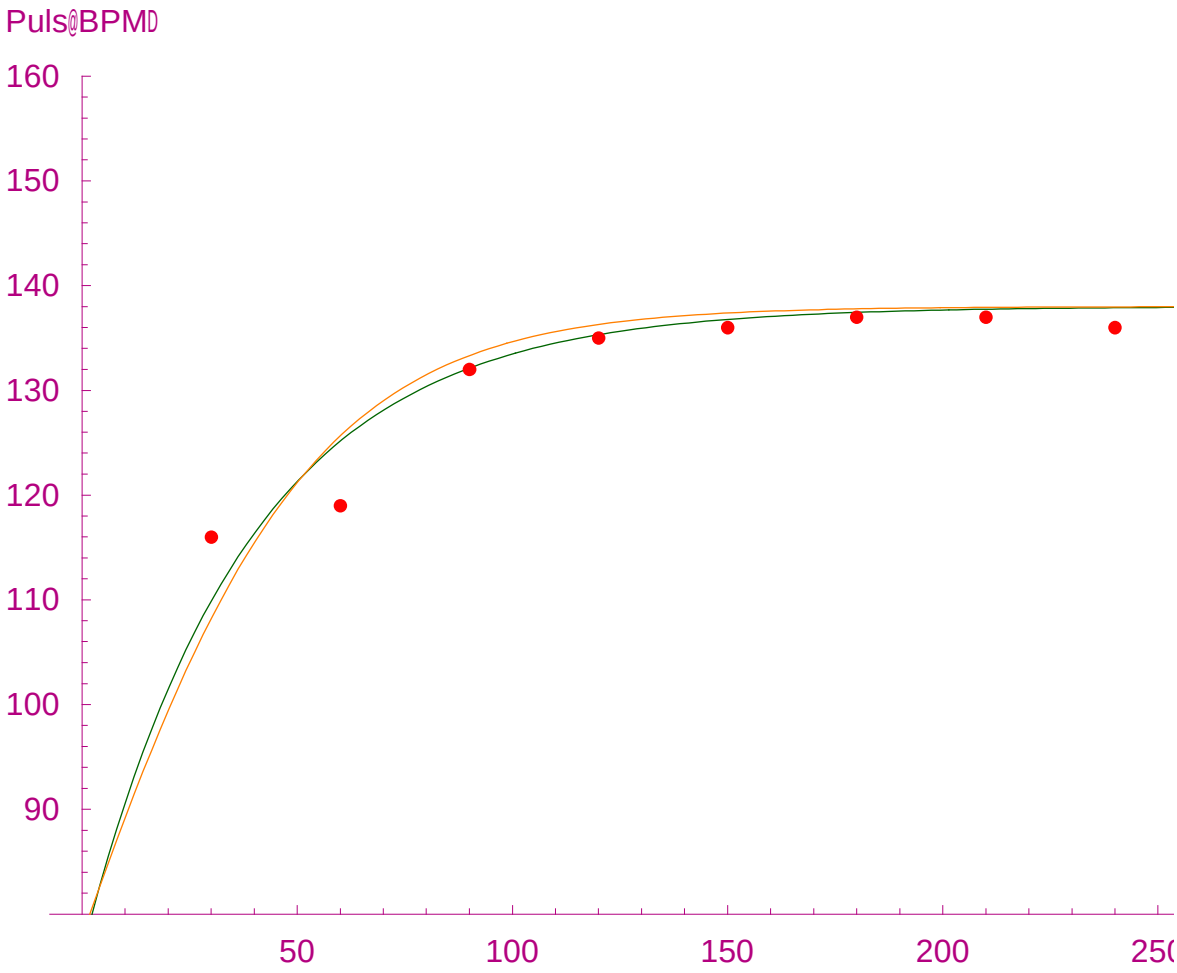
Comparison of the districted and logistical growth first trial:

Open / Close

1. Thomas Spath

In this graph you can see the optical difference between the fitted restricted and logistical growth. For this trial also the sum of squared errors was nearly the same.

```
Plot[{138 - 61.53365064137114` 0.9742422021445735`^x,
      138 / (1 + 0.7716196511695537` e^(-0.03433971146888444` x))}, {x, 0, 300},
 PlotRange → {160, 80},
 PlotStyle → {DarkGreen, Orange, Thickness[0.01]},
 Epilog → {Red, PointSize[0.01], Point /@ trial1tsres},
 AxesLabel → {"Zeit[sec]", "Puls[BPM]"}];
```



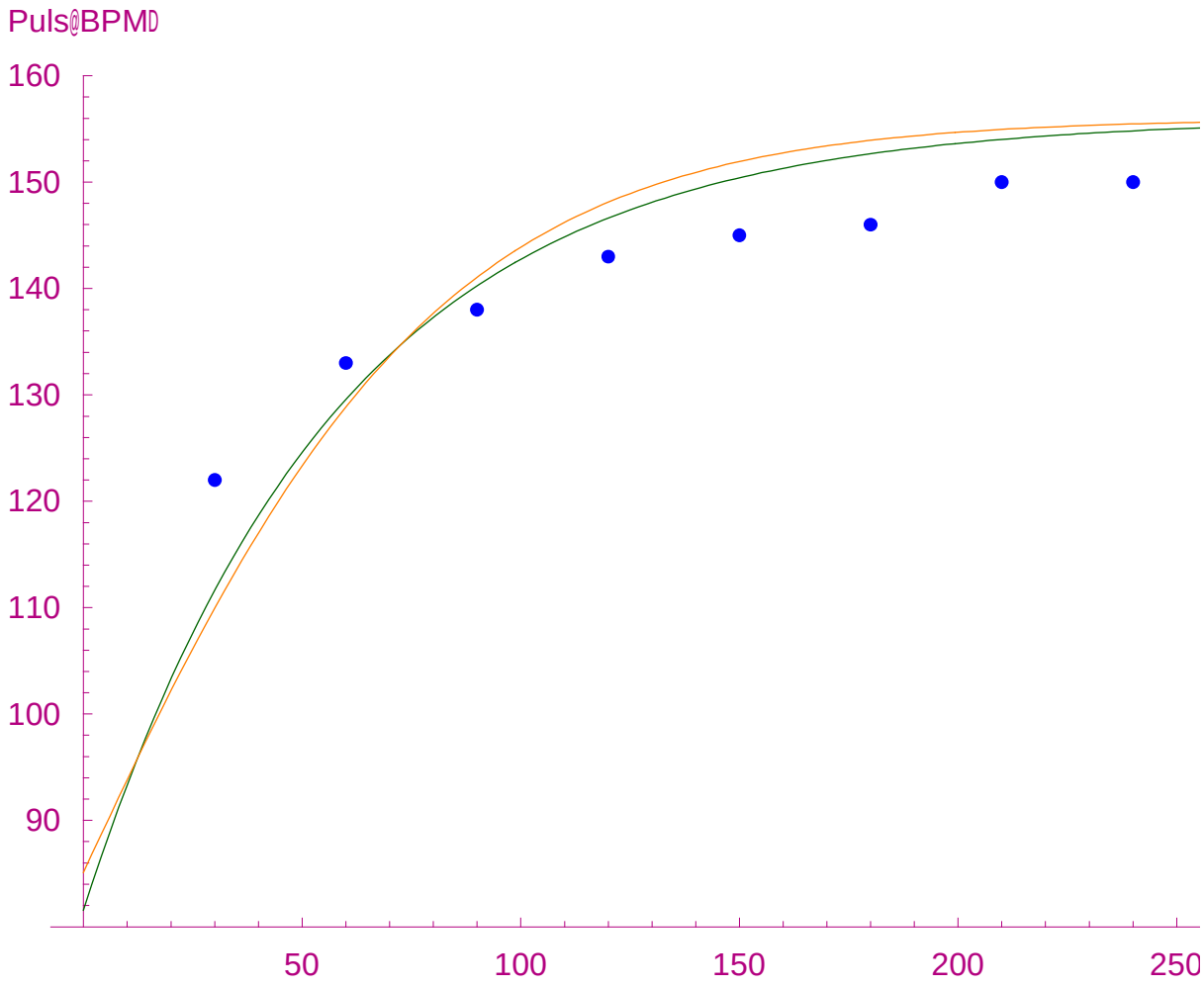
2. Christian Großschädl

In this graph you can see the optical difference between the fitted restricted and logistical growth. For this trial also the sum of squared errors was nearly the same.

Input ▷

```
Plot[{156 - 74.44796464387261` 0.9828893041670148`^x,
      156 / (1 + 0.8330368618408186` e^(-0.022915819875373118` x))}, {x, 0, 300},
 PlotRange → {160, 80},
```

```
PlotStyle → { DarkGreen, Orange, Thickness[0.01]},
Epilog → {Blue, PointSize[0.01], Point /@ trial1cgres},
AxesLabel → {"Zeit[sec]", "Puls[BPM]"}];
```



Comparison of the districted and logistical growth second trial:

Open / Close

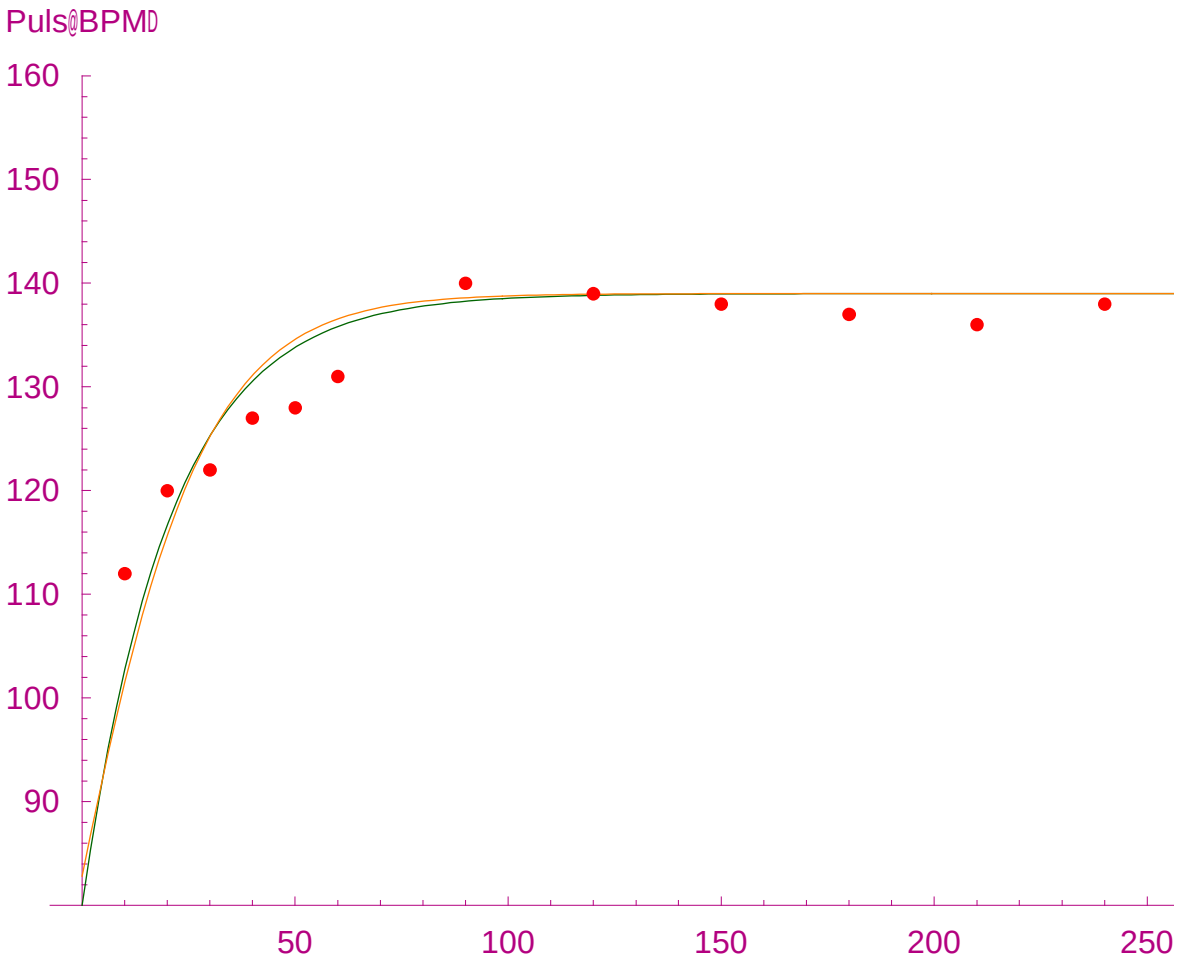
3. Thomas Spath

In this graph you can see the optical difference between the fitted restricted and logistical growth. For this trial also the sum of squared errors was nearly the same.

Input ▷

```
Plot[{139 - 59.033330370942835` 0.9525551234586229`^x,
                    139
 1 + 0.6782546334322301` e^-0.0605964340362175` x}, {x, 0, 300},
PlotRange → {160, 80},
```

```
    PlotStyle → {DarkGreen, Orange, Thickness[0.01]},
    Epilog → {Red, PointSize[0.01], Point /@ trial2tsres},
    AxesLabel → {"Zeit[sec]", "Puls[BPM]"}];
```

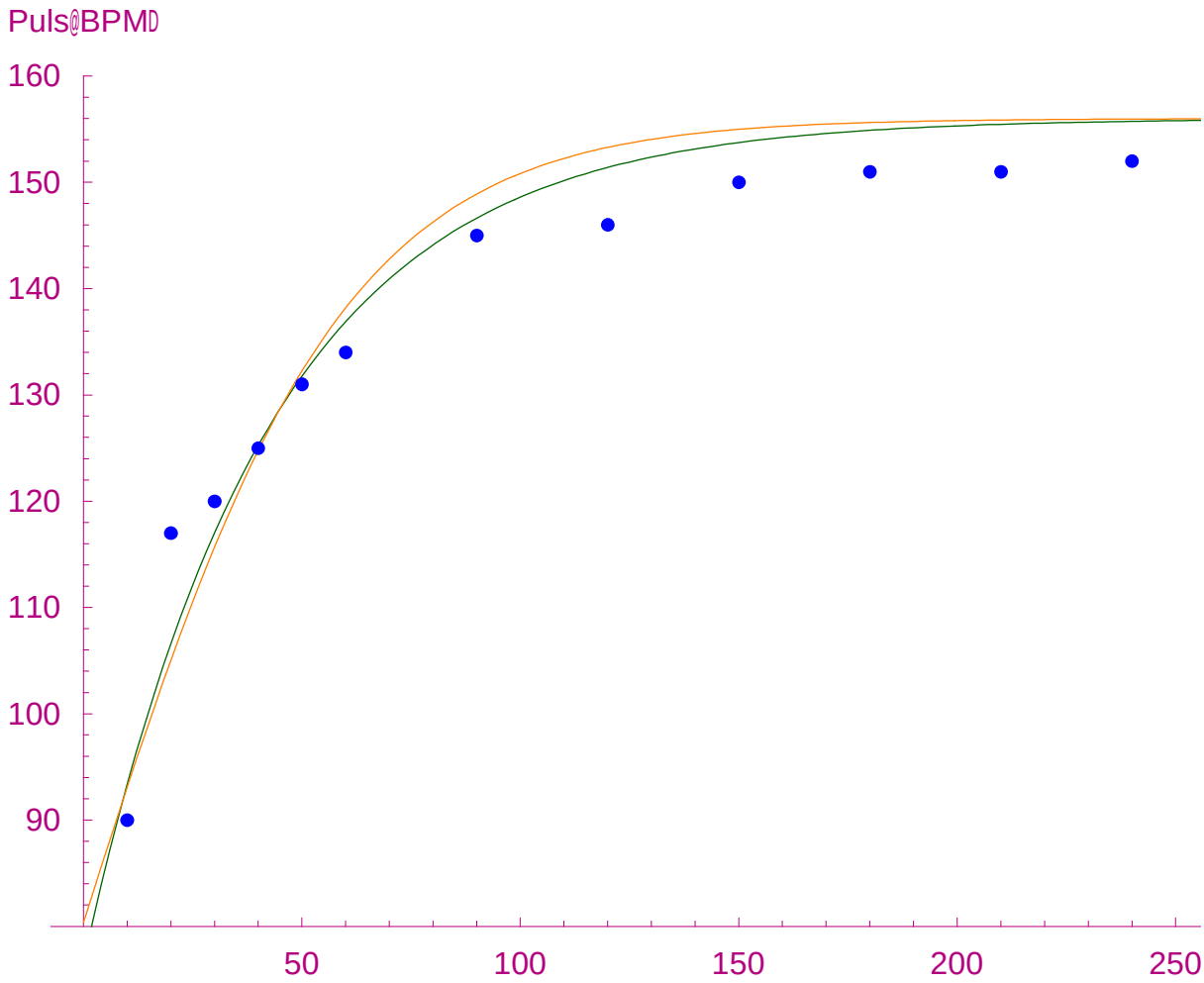


4. Christian Großschädl

In this graph you can see the optical difference between the fitted restricted and logistical growth. For this trial also the sum of squared errors was nearly the same.

Input ▷

```
Plot[{156 - 79.4345815707685` 0.9765448416655254`^x,
                    156
  ------------------------------------------}, {x, 0, 300},
  1 + 0.9416396426961438` e^-0.033129086285494054` x
  PlotRange → {160, 80},
  PlotStyle → { DarkGreen, Orange, Thickness[0.01]},
  Epilog → {Blue, PointSize[0.01], Point /@ trial2cgres},
  AxesLabel → {"Zeit[sec]", "Puls[BPM]"}];
```



Comparison of the linear, logistical and exponential decay second trial:

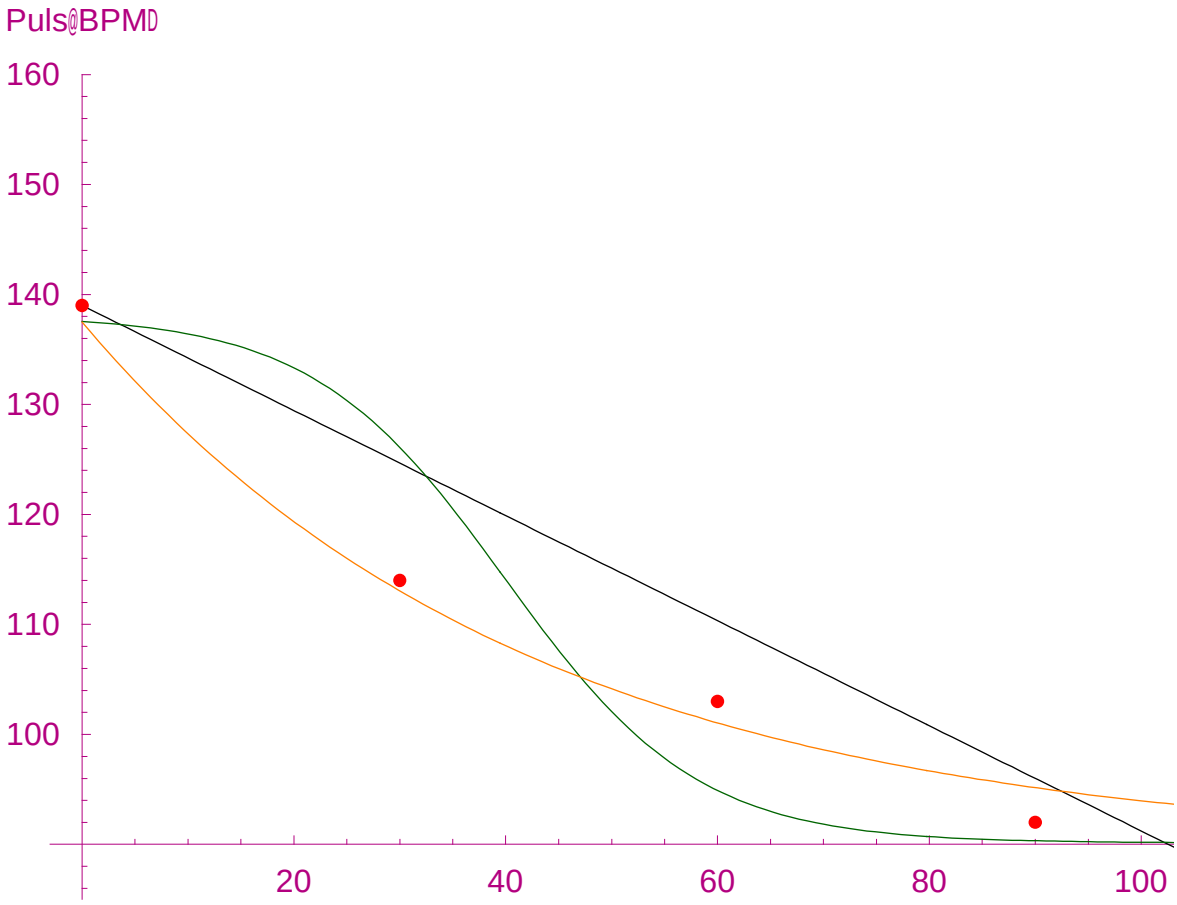
Open / Close

5. Thomas Spath

This graph shows a comparison of the fitted funtions for the regeneraton phase of the second trial. As you can see only the exponential decay gave us a good looking result.

Input ▹

```
Plot[{139 - 0.4777777777777779` x,
  90.12929176275199` + 48/(1 + 0.01234` e^(0.11` x)),
  89.48857896795404` + 48 e^(-0.02373456954195246` x)}, {x, 0, 120},
 PlotRange → {160, 85},
 PlotStyle → { Black, DarkGreen, Orange, Thickness[0.01]},
 Epilog → {Red, PointSize[0.01], Point /@ trial2tsreg},
 AxesLabel → {"Zeit[sec]", "Puls[BPM]"}];
```

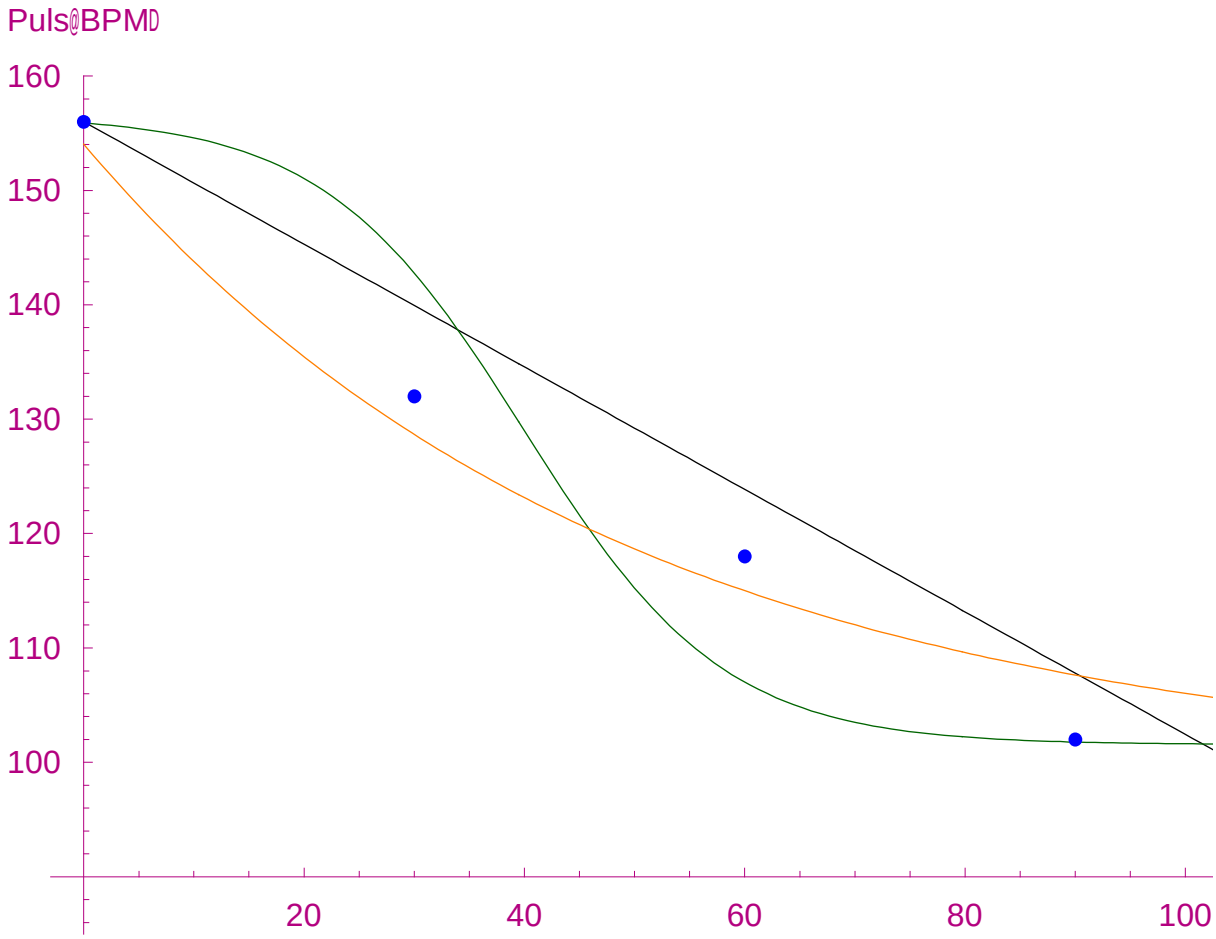


6. Christian Großschädl

This graph shows a comparison of the fitted funtions for the regeneraton phase of the second trial. As you can see only the exponential decay gave us a good looking result.

Input ▷

```
Plot[{156 - 0.5355555555555557` x,
  101.552252238488` + 55/(1 + 0.01234` E^(0.11` x)),
  99.0359727667226` + 55 E^(-0.020619617027890766` x)}, {x, 0, 120},
 PlotRange → {160, 85},
 PlotStyle → { Black, DarkGreen, Orange, Thickness[0.01]},
 Epilog → {Blue, PointSize[0.01], Point /@ trial2cgreg},
 AxesLabel → {"Zeit[sec]", "Puls[BPM]"}];
```



Comparison of the districted and logistical growth third trial:

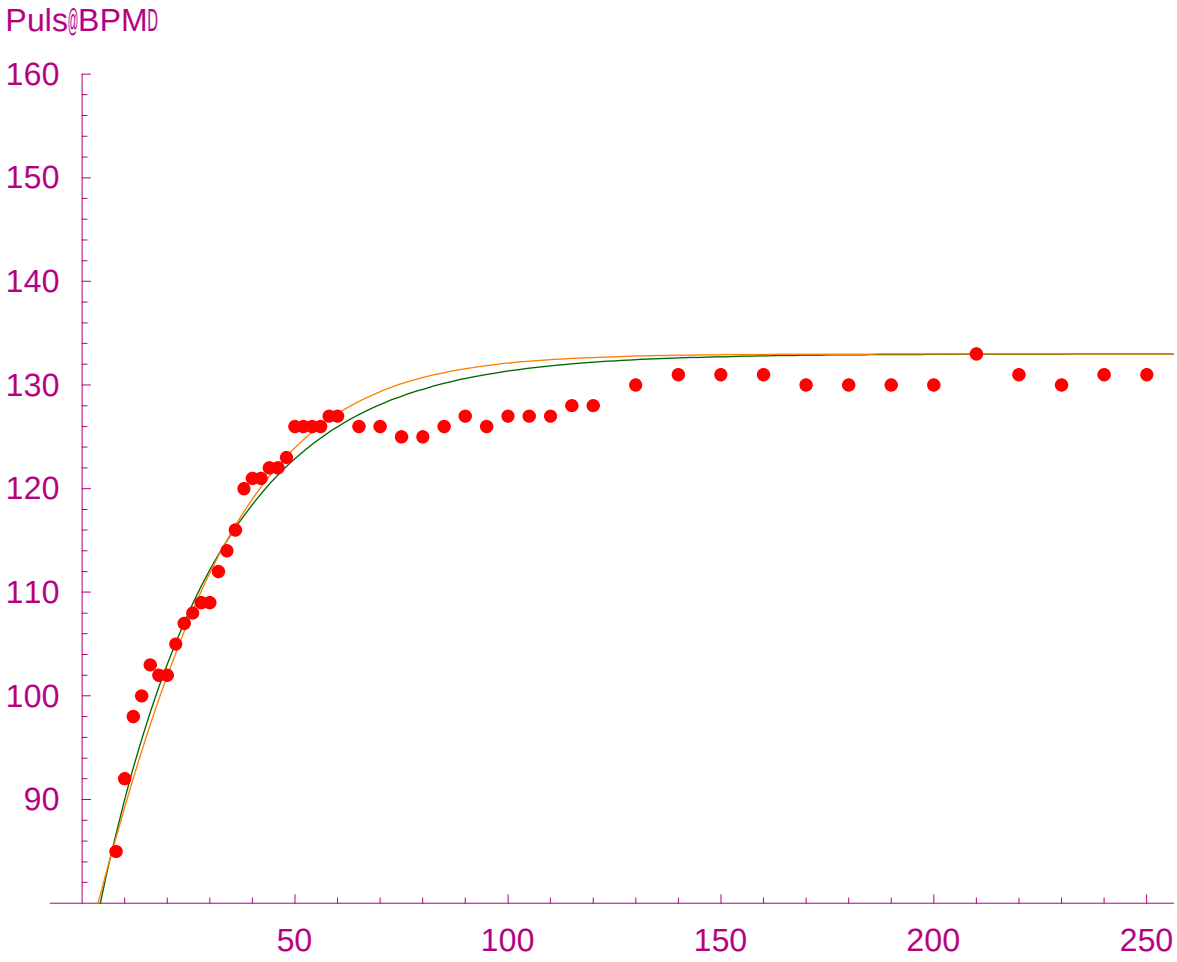
Open / Close

7. Thomas Spath

The data of the third trial brought a bigger difference, at least in the sum of squared errors, favouring the restricted growth.

Input ▹

```
Plot[{133 - 61.81640455244227` 0.9644089495759743`^x,
   133/(1 + 0.790131069118365` e^(-0.04765250744413003` x))}, {x, 0, 300},
  PlotRange → {160, 80},
  PlotStyle → {DarkGreen, Orange, Thickness[0.01]},
  Epilog → {Red, PointSize[0.01], Point /@ trial3tsres},
  AxesLabel → {"Zeit[sec]", "Puls[BPM]"}];
```

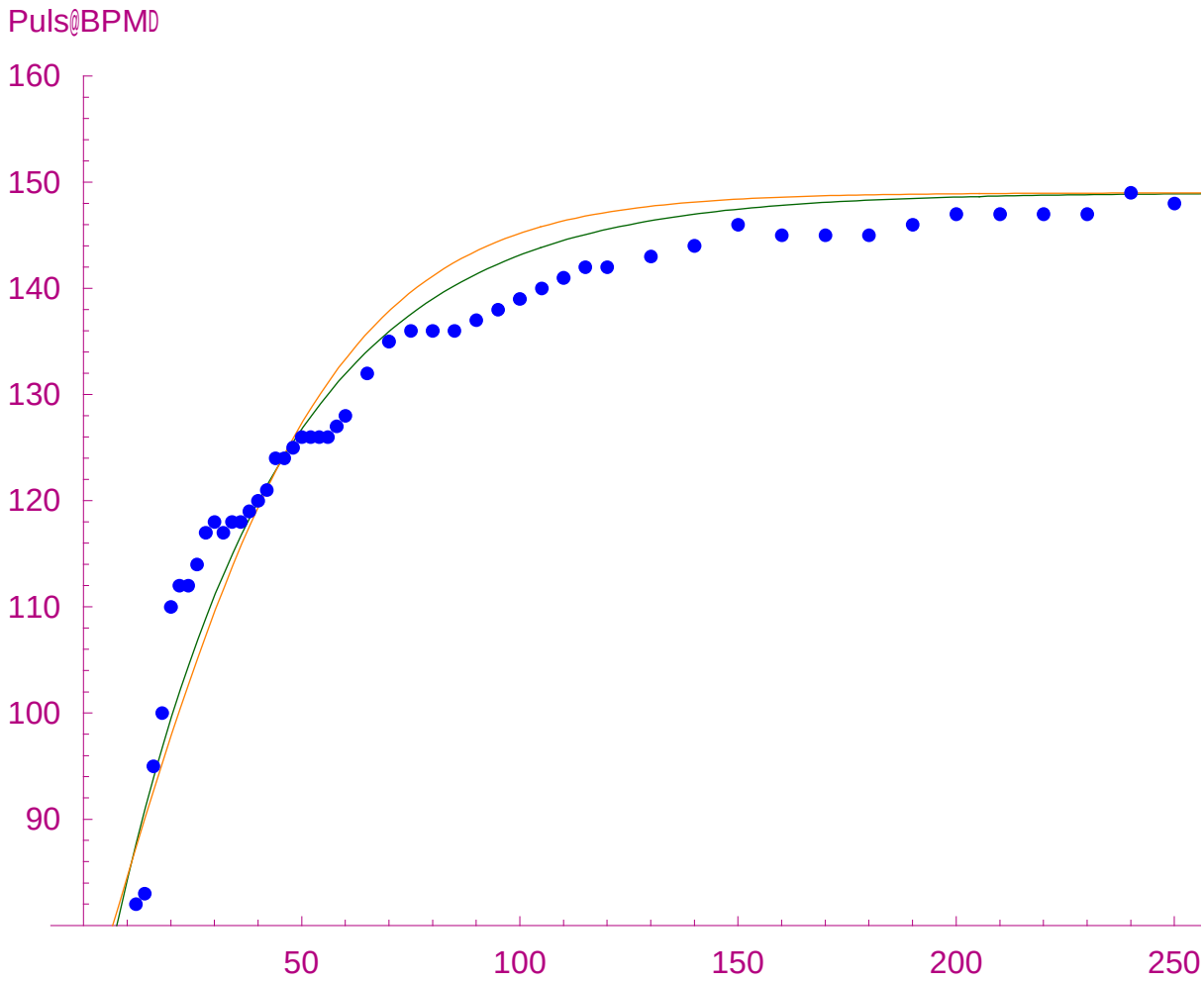


8. Christian Großschädl

The data of the third trial brought a bigger difference, at least in the sum of squared errors, favouring the restricted growth.

Input ▹

```
Plot[{149 - 84.60033226942635` 0.9736652175467784`^x,
   149/(1 + 1.108152522135262` E^(-0.03740389438921834` x))}, {x, 0, 300},
  PlotRange → {160, 80},
  PlotStyle → {DarkGreen, Orange, Thickness[0.01]},
  Epilog → {Blue, PointSize[0.01], Point /@ trial3cgres},
  AxesLabel → {"Zeit[sec]", "Puls[BPM]"}];
```

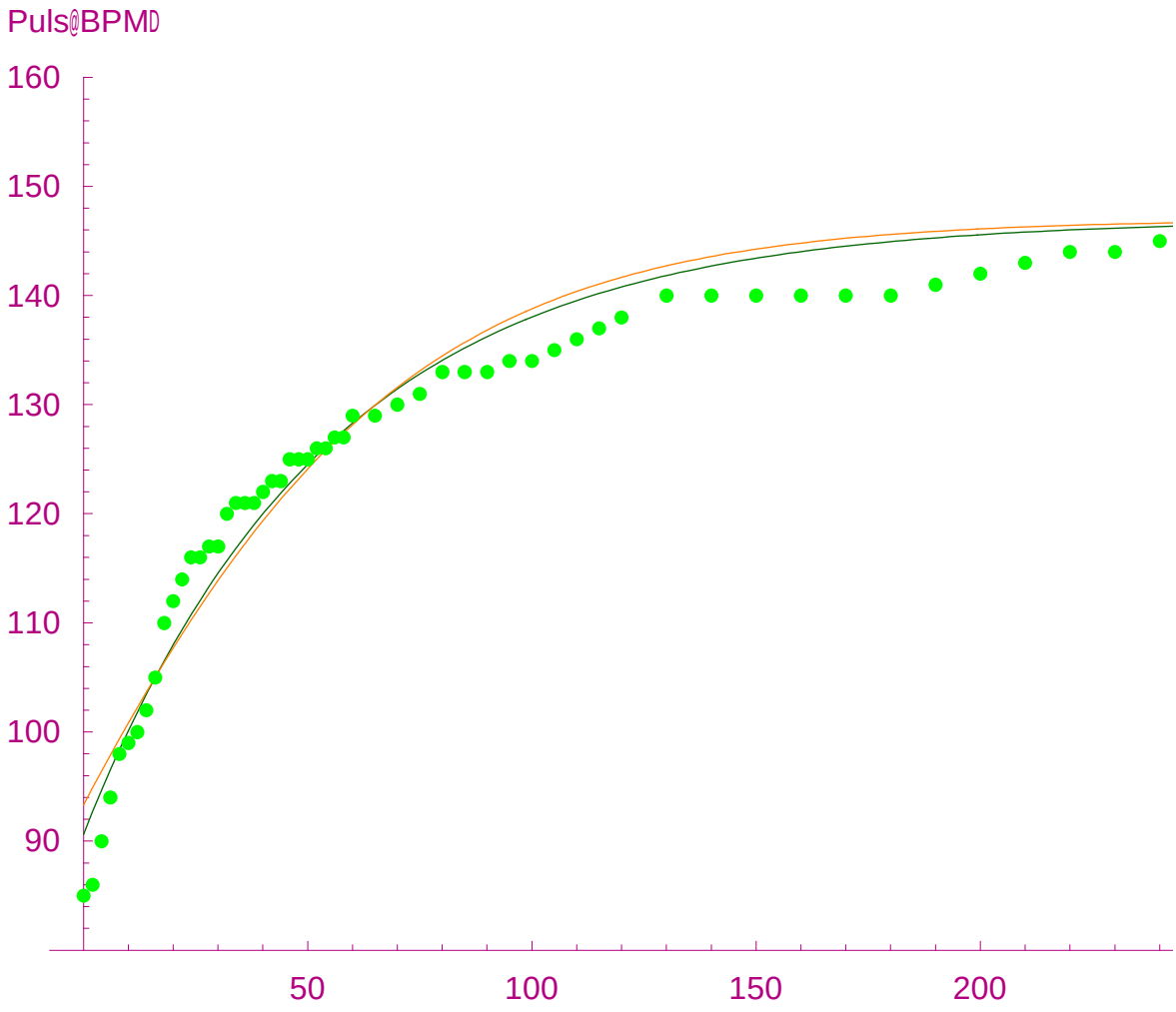


9. Christoph Spath

The data of the third trial brought a bigger difference, at least in the sum of squared errors, favouring the restricted growth.

Input ▻

```
Plot[{147 - 56.35531624945529` 0.9817864772189573`^x,
   147/(1 + 0.5751229287107462` E^(-0.022733433062745008` x))}, {x, 0, 300},
  PlotRange → {160, 80},
  PlotStyle → { DarkGreen, Orange, Thickness[0.01]},
  Epilog → {Green, PointSize[0.01], Point /@ trial3csres},
  AxesLabel → {"Zeit[sec]", "Puls[BPM]"}];
```



Comparison of the linear, logistical and exponential decay third trial:

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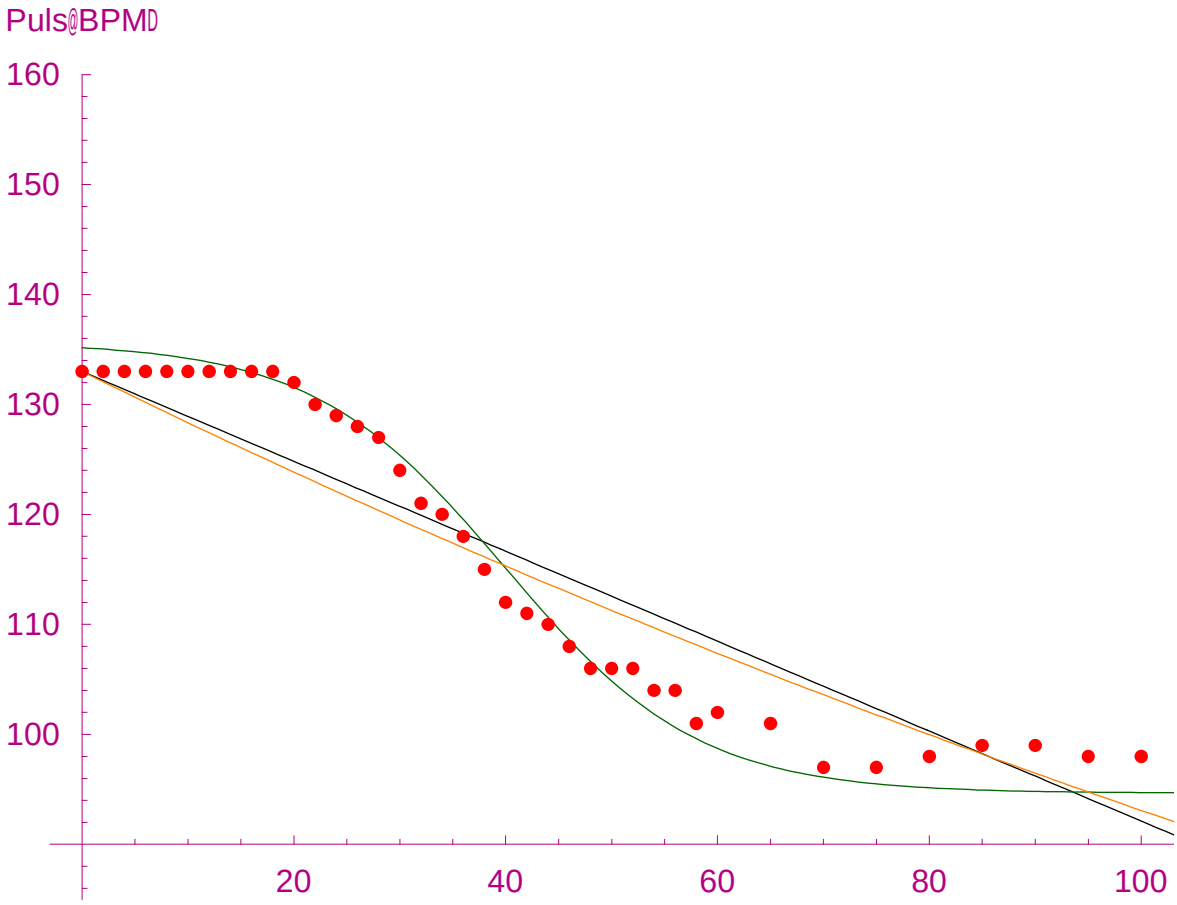
10. Thomas Spath

The fit of the third trial's regeneration brought, in case of Thomas Spath, a astonishing result with an error of only 2.15 per data point.

Input ▷

```
Plot[{133 - 0.4088082182272507` x,
  94.65128861542226` + 41/(1 + 0.01234` e^(0.11` x)),
  133 e^(-0.003568220595670958 3` x)}, {x, 0, 120},
 PlotRange → {160, 85},
 PlotStyle → {Black, DarkGreen, Orange, Thickness[0.01]},
 Epilog → {Red, PointSize[0.01], Point /@ trial3tsreg},
 AxesLabel → {"Zeit[sec]", "Puls[BPM]"}];
```

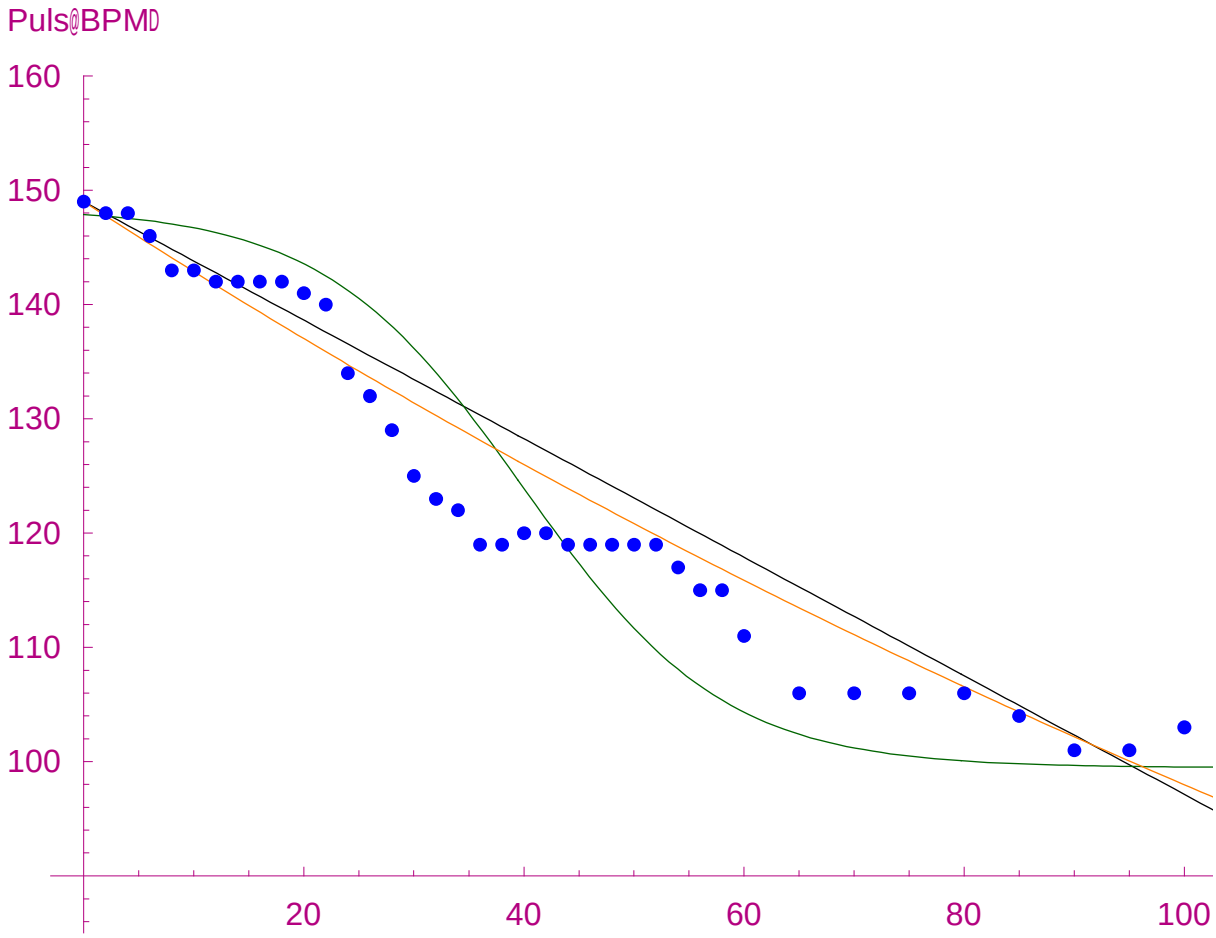


11. Christian Großschädl

The fit for Christians values brought us not any satisfying result as you can see.

Input ▹

```
Plot[{149 - 0.5185951273686403` x,
  99.4664010849091` + 49/(1 + 0.01234` ℯ^(0.11` x)),
  149 ℯ^(-0.004191558538275857` x)}, {x, 0, 120},
 PlotRange → {160, 85},
 PlotStyle → {Black, DarkGreen, Orange, Thickness[0.01]},
 Epilog → {Blue, PointSize[0.01], Point /@ trial3cgreg},
 AxesLabel → {"Zeit[sec]", "Puls[BPM]"}];
```

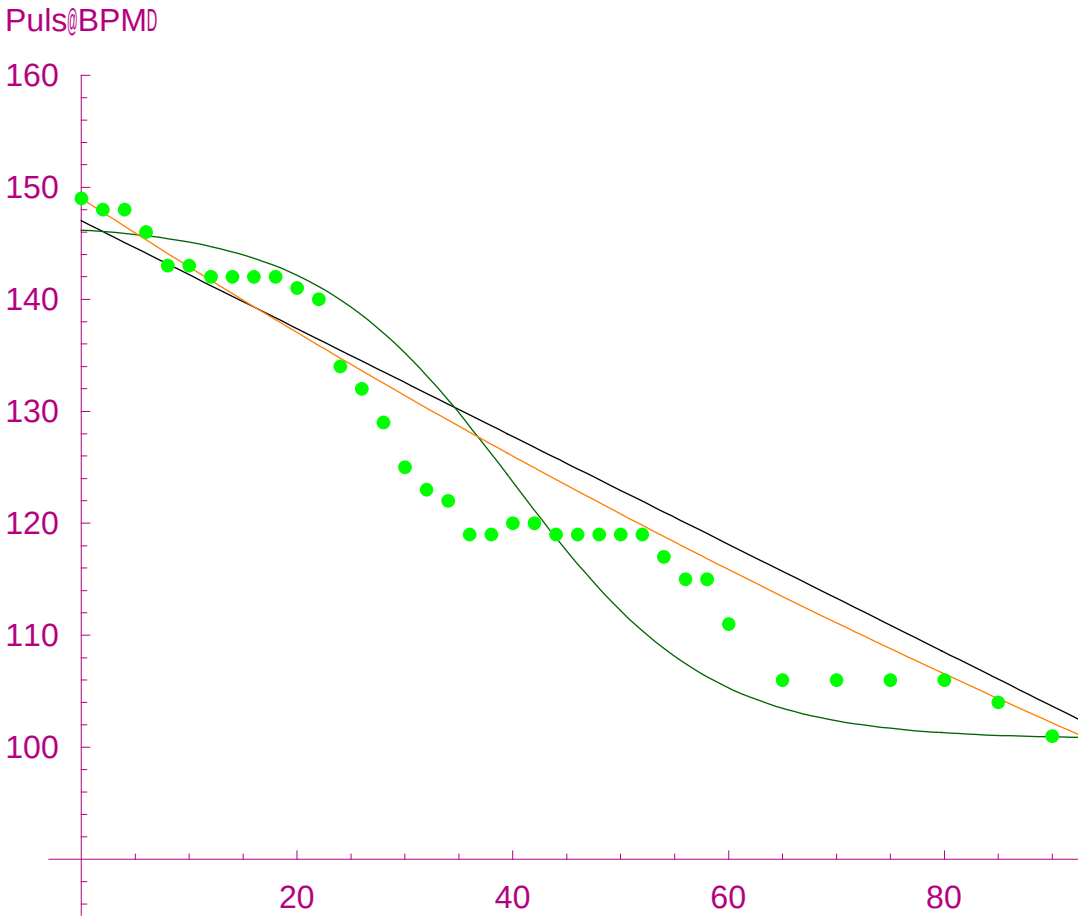


12. Christoph Spath

The same applies to the results of Christoph Spath.

Input ▷

```
Plot[{147 - 0.48139793156104677` x,
  100.73922228094457` + 46/(1 + 0.01234` e^(0.11` x)),
  149 e^(-0.004191558538275857` x)}, {x, 0, 120},
 PlotRange → {160, 85},
 PlotStyle → {Black, DarkGreen, Orange, Thickness[0.01]},
 Epilog → {Green, PointSize[0.01], Point /@ trial3cgreg},
 AxesLabel → {"Zeit[sec]", "Puls[BPM]"}];
```



Prognosis:

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In this chapter we take our best fitting functions and try to make prognosis with them.Additionally we compare some values.

13. When did we reach 100 BPM

We chose 100 BPM because it seemed to be a significant point in the graphs and it also is the first three digit value.

14. Thomas Spath

Input ▹
```
f[x_] = 133 - 61.81640455244227` 0.9644089495759743`^x;
```

The Sum of squared errors from this function was 496.631, the average error per point was 3.18288 BPM.

Input ▹
```
NSolve[f[x] == 100]
```

```
{{x → 17.3196}}
```

Thomas made the second place with keeping his heartrate 17 seconds below 100 BPM.

15. Christian Großschädl

Input ▹
```
f[x_] = 149 - 84.60033226942635` 0.9736652175467784`^x;
```

The Sum of squared errors from this function was 992,5 the average error per point was 4.03367 BPM.

Input ▹
```
NSolve[f[x] == 100]
```

```
{{x → 20.4632}}
```

Christian made it, he won this "competition" with 20 seconds below 100 BPM

16. Christoph Spath

Input ▹
```
f[x_] = 147 - 56.35531624945529` 0.9817864772189573`^x;
```

The Sum of squared errors from this function was 533.18, the average error per point was 2.95646 BPM.

Input ▹
```
NSolve[f[x] == 100]
```

```
{{x → 9.87567}}
```

Christoph ran far behind the others with only 9.8 seconds below 100 BPM.

17. Who did reach 95% of his maximum heartrate at first?

We decided to make another prognosis as we wanted to know how long every proband needed to reach 95 % of his max. heartrate. We chose this value due to the fact that after that value the heartrate didn't change much anymore.

18. Thomas Spath

Input ▹
```
f[x_] = 133 - 61.81640455244227` 0.9644089495759743`^x;
```

Input ▹
```
NSolve[f[x] == 0.95 * trial3tsresmax]
```

```
{{x → 61.5221}}
```

As you can see, Thomas reached the 95 % in first.

19. Christian Großschädl

Input ▹
```
f[x_] = 149 - 84.60033226942635` 0.9736652175467784`^x;
```

Input ▹
```
NSolve[f[x] == 0.95 * trial3cgresmax]
```

```
{{x → 91.0427}}
```

The second place goes to Christian who has a gap of 30 seconds to Thomas.

20. Christoph Spath

Input ▹
```
f[x_] = 147 - 56.35531624945529` 0.9817864772189573`^x;
```

Input ▹
```
NSolve[f[x] == 0.95 * trial3csresmax]
```

```
{{x → 110.817}}
```

...And the winner is Christoph who kept his heartrate 110 seconds below the 95 % mark.

5 Our Team

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Participants

Open / Close

1. Thomas Spath

Born: 21.11.1988
Schools: Elementary School Ragnitz, grammar school Leibnitz, HTC Arnfels (mechatronics, left after 3 years)

Current School: Commercial highschool Leibnitz (4^{th} grade)

Tasks: "scientist",proband
E-Mail adress: thomas.spath88@gmail.com

2. Christian Großschädl

Born: 05.09.1991
Schools: Elementary School Wolfsberg, grammar school Leibnitz
Current School: Commercial highschool Leibnitz (4^{th} grade)

Tasks: "scientist",proband
E-Mail adress: christian_grossschaedl@hotmail.com

Christoph Spath

Born: 25.12.1989
Schools: Elementary School Ragnitz, secondary modern school of music Großklein
Current School: grammar school of music Dreihackengasse Graz (4^{th} grade)
Tasks: proband

4. Uta Geratz

Born: 3.10.1990
Schools: Christoph-Rensing-Schule Dormagen-Horrem,
Bettina-von-Arnim-Gymnasium Dormagen
Current School: Bettina-von-Arnim-Gymnasium
Tasks:"scientist"

5. Special thanks to Prof. Mag. Breivogel and our advisor Mag. Wolfgang Narrath.

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